



Special relativity and steps towards general relativity: SR

(c) CC-BY-SA-3.0





SR+GR

- SR+GR: construct spacetime from set theory





SR+GR

- SR+GR: construct spacetime from set theory
- start with X = set of points, no distances between points, angles, etc. defined





SR+GR

- SR+GR: construct spacetime from set theory
- start with X = set of points, no distances between points, angles, etc. defined
- no absolute simultaneity = “time is not absolute”





SR+GR

- SR+GR: construct spacetime from set theory
- start with X = set of points, no distances between points, angles, etc. defined
- no absolute simultaneity = “time is not absolute”
- add properties to X that satisfy theorems





SR+GR

- SR+GR: construct spacetime from set theory
- start with X = set of points, no distances between points, angles, etc. defined
- no absolute simultaneity = “time is not absolute”
- add properties to X that satisfy theorems
- → differentiable 4-(pseudo-)manifold





SR+GR

- SR+GR: construct spacetime from set theory
- start with X = set of points, no distances between points, angles, etc. defined
- no absolute simultaneity = “time is not absolute”
- add properties to X that satisfy theorems
- → differentiable 4-(pseudo-)manifold
- point particle in space → w:World line in spacetime





SR+GR

- SR+GR: construct spacetime from set theory
- start with X = set of points, no distances between points, angles, etc. defined
- no absolute simultaneity = “time is not absolute”
- add properties to X that satisfy theorems
- → differentiable 4-(pseudo-)manifold
- point particle in space → w:World line in spacetime
- point in spacetime → spacetime “event”





SR+GR

- SR+GR: construct spacetime from set theory
- start with X = set of points, no distances between points, angles, etc. defined
- no absolute simultaneity = “time is not absolute”
- add properties to X that satisfy theorems
- → differentiable 4-(pseudo-)manifold
- point particle in space → w:World line in spacetime
- point in spacetime → spacetime “event”
- SR: spacetime = w:Minkowski space





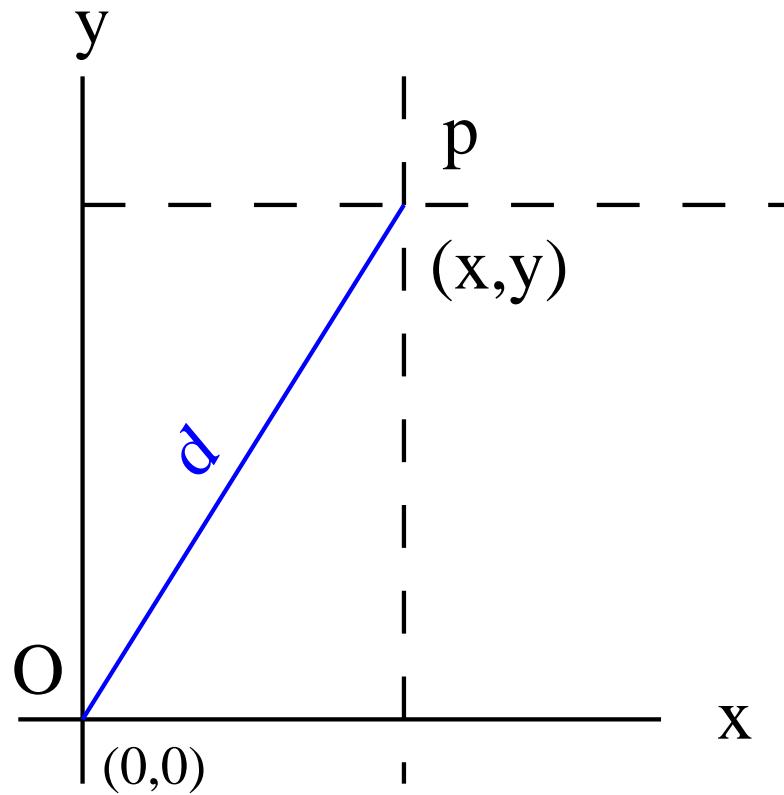
SR+GR

- SR+GR: construct spacetime from set theory
- start with X = set of points, no distances between points, angles, etc. defined
- no absolute simultaneity = “time is not absolute”
- add properties to X that satisfy theorems
- → differentiable 4-(pseudo-)manifold
- point particle in space → w:World line in spacetime
- point in spacetime → spacetime “event”
- SR: spacetime = w:Minkowski space
- GR: spacetime = a solution of the
w:Einstein field equations





SR: Minkowski spacetime

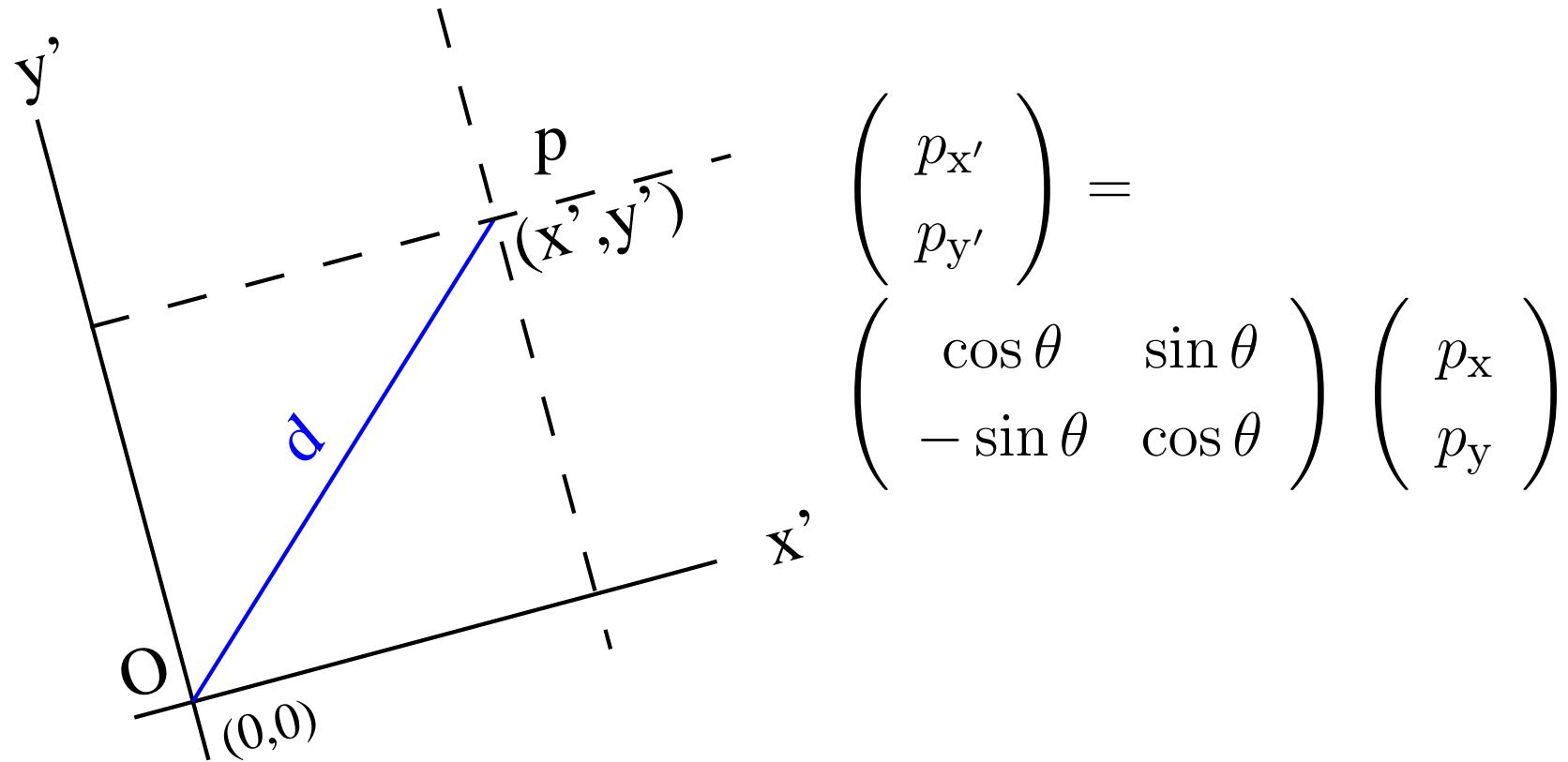


p at (x, y) , distance from observer at O is d





SR: Minkowski spacetime

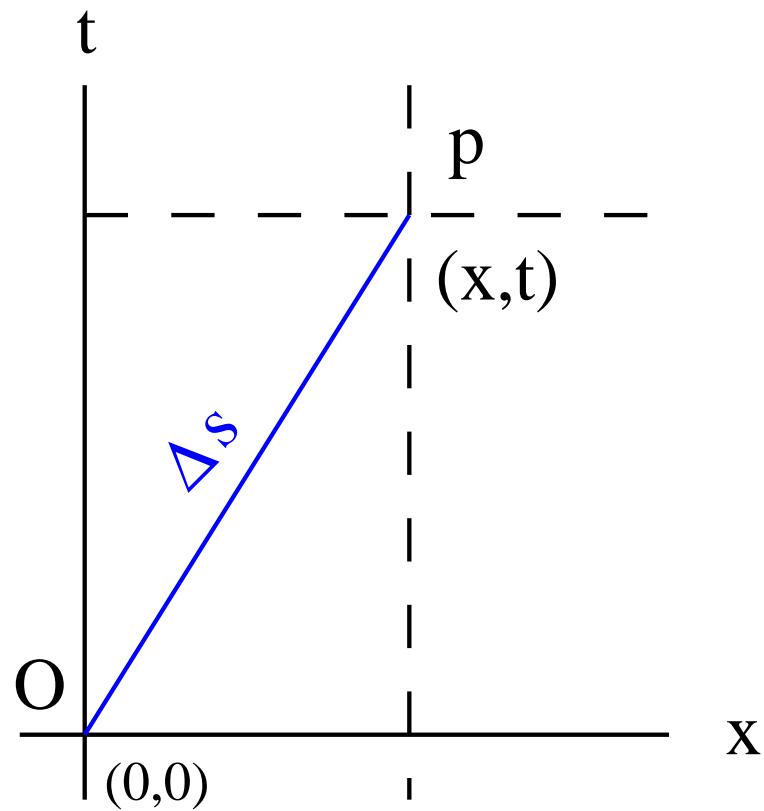


p at (x', y') , distance from observer at O is d = unchanged





SR: Minkowski spacetime

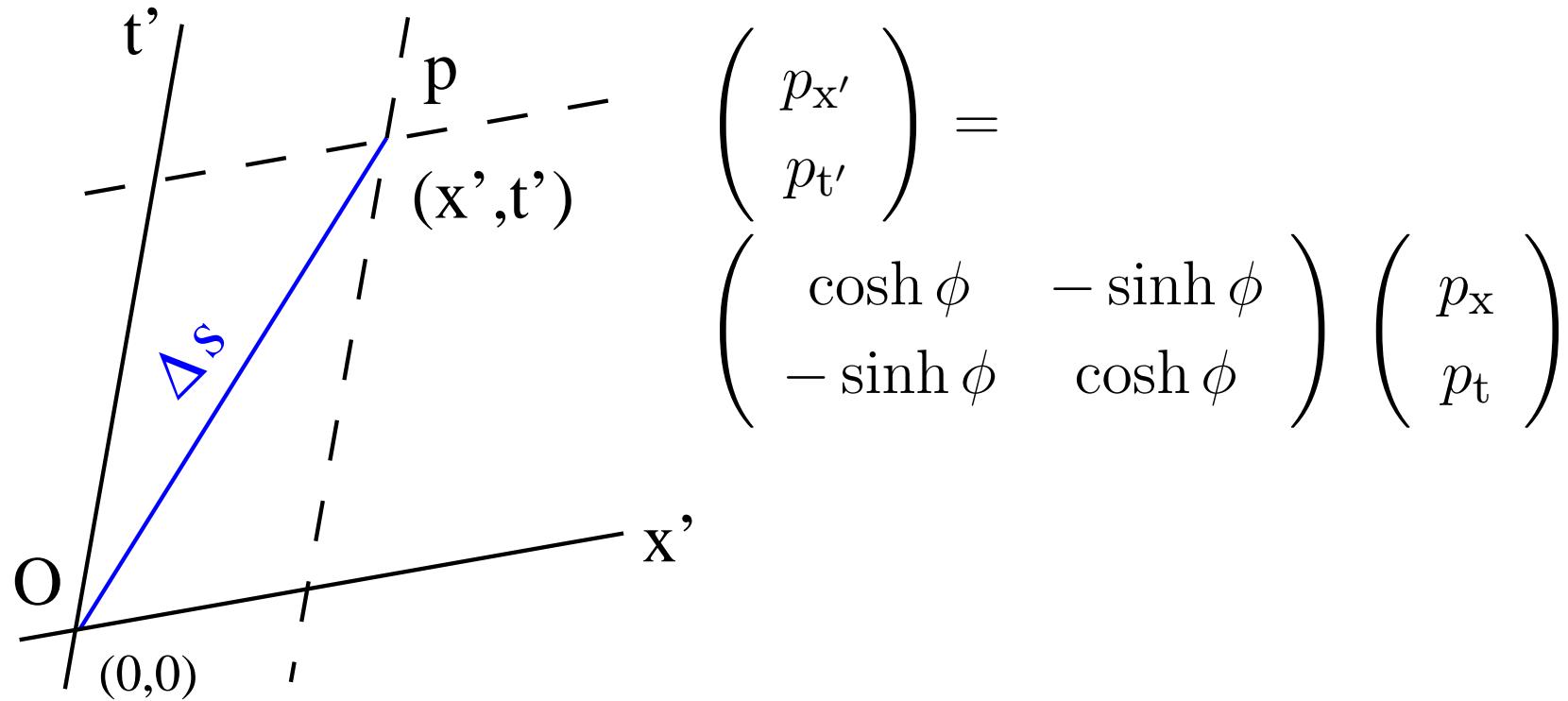


p at (x, t) , w:invariant interval from observer at O is Δs
where $(\Delta s)^2 = -(\Delta t)^2 + (\Delta x)^2$





SR: Minkowski spacetime



p at (x', t') , invariant interval from observer at O is $\Delta s = (\Delta s)^2 = -(\Delta t')^2 + (\Delta x')^2 = \text{unchanged}$





SR: Lorentz transformation

$$\Lambda(\phi) := \begin{pmatrix} \cosh \phi & -\sinh \phi \\ -\sinh \phi & \cosh \phi \end{pmatrix}$$

$$\cosh \phi := \frac{e^\phi + e^{-\phi}}{2}$$

$$\sinh \phi := \frac{e^\phi - e^{-\phi}}{2}$$

w:hyperbolic function





SR: Lorentz transformation

$$\Lambda(\phi) := \begin{pmatrix} \cosh \phi & -\sinh \phi \\ -\sinh \phi & \cosh \phi \end{pmatrix}$$

$$\cosh \phi := \frac{e^\phi + e^{-\phi}}{2}$$

$$\sinh \phi := \frac{e^\phi - e^{-\phi}}{2}$$

w:hyperbolic function

$$\begin{pmatrix} p_{x'} \\ p_{t'} \end{pmatrix} = \Lambda \begin{pmatrix} p_x \\ p_t \end{pmatrix}$$





SR: Lorentz transformation

$$\Lambda(\phi) := \begin{pmatrix} \cosh \phi & -\sinh \phi \\ -\sinh \phi & \cosh \phi \end{pmatrix}$$

$$\cosh \phi := \frac{e^\phi + e^{-\phi}}{2}$$

$$\sinh \phi := \frac{e^\phi - e^{-\phi}}{2}$$

w:hyperbolic function

$$\begin{pmatrix} p_{x'} \\ p_{t'} \end{pmatrix} = \Lambda \begin{pmatrix} p_x \\ p_t \end{pmatrix}$$

units: Λ only makes sense if same units for x, t, x', t'





SR: Lorentz transformation

$$\Lambda(\phi) := \begin{pmatrix} \cosh \phi & -\sinh \phi \\ -\sinh \phi & \cosh \phi \end{pmatrix}$$

$$\cosh \phi := \frac{e^\phi + e^{-\phi}}{2}$$

$$\sinh \phi := \frac{e^\phi - e^{-\phi}}{2}$$

w:hyperbolic function

$$\begin{pmatrix} p_{x'} \\ p_{t'} \end{pmatrix} = \Lambda \begin{pmatrix} p_x \\ p_t \end{pmatrix}$$

units: Λ only makes sense if same units for x, t, x', t'

Definition: $1 \text{ m} := (1/2.99792458 \times 10^8) \text{ s} \approx 10^{-8.5} \text{ s} \approx 3 \text{ ns}$





SR: Lorentz transformation

$$\Lambda(\phi) := \begin{pmatrix} \cosh \phi & -\sinh \phi \\ -\sinh \phi & \cosh \phi \end{pmatrix}$$

$$\cosh \phi := \frac{e^\phi + e^{-\phi}}{2}$$

$$\sinh \phi := \frac{e^\phi - e^{-\phi}}{2}$$

w:hyperbolic function

$$\begin{pmatrix} p_{x'} \\ p_{t'} \end{pmatrix} = \Lambda \begin{pmatrix} p_x \\ p_t \end{pmatrix}$$

units: Λ only makes sense if same units for x, t, x', t'

Definition: $1 \text{ m} := (1/2.99792458 \times 10^8) \text{ s} \approx 10^{-8.5} \text{ s} \approx 3 \text{ ns}$

speed of light in a vacuum = $c = \frac{2.99792458 \times 10^8 \text{ m}}{1 \text{ s}}$





SR: Lorentz transformation

$$\Lambda(\phi) := \begin{pmatrix} \cosh \phi & -\sinh \phi \\ -\sinh \phi & \cosh \phi \end{pmatrix}$$

$$\cosh \phi := \frac{e^\phi + e^{-\phi}}{2}$$

$$\sinh \phi := \frac{e^\phi - e^{-\phi}}{2}$$

w:**hyperbolic function**

$$\begin{pmatrix} p_{x'} \\ p_{t'} \end{pmatrix} = \Lambda \begin{pmatrix} p_x \\ p_t \end{pmatrix}$$

units: Λ only makes sense if same units for x, t, x', t'

Definition: $1 \text{ m} := (1/2.99792458 \times 10^8) \text{ s} \approx 10^{-8.5} \text{ s} \approx 3 \text{ ns}$

speed of light in a vacuum = $c = \frac{2.99792458 \times 10^8 \text{ m}}{1 \text{ s}}$

L = $\frac{2.99792458 \times 10^8 \times (1/2.99792458 \times 10^8) \text{ s}}{1 \text{ s}}$



SR: Lorentz transformation

$$\Lambda(\phi) := \begin{pmatrix} \cosh \phi & -\sinh \phi \\ -\sinh \phi & \cosh \phi \end{pmatrix}$$

$$\cosh \phi := \frac{e^\phi + e^{-\phi}}{2}$$

$$\sinh \phi := \frac{e^\phi - e^{-\phi}}{2}$$

w:**hyperbolic function**

$$\begin{pmatrix} p_{x'} \\ p_{t'} \end{pmatrix} = \Lambda \begin{pmatrix} p_x \\ p_t \end{pmatrix}$$

units: Λ only makes sense if same units for x, t, x', t'

Definition: $1 \text{ m} := (1/2.99792458 \times 10^8) \text{ s} \approx 10^{-8.5} \text{ s} \approx 3 \text{ ns}$

speed of light in a vacuum = $c = \frac{2.99792458 \times 10^8 \text{ m}}{1 \text{ s}}$

$$\text{L} = \frac{1 \text{ s}}{1 \text{ s}}$$



SR: Lorentz transformation

$$\Lambda(\phi) := \begin{pmatrix} \cosh \phi & -\sinh \phi \\ -\sinh \phi & \cosh \phi \end{pmatrix}$$

$$\cosh \phi := \frac{e^\phi + e^{-\phi}}{2}$$

$$\sinh \phi := \frac{e^\phi - e^{-\phi}}{2}$$

w:**hyperbolic function**

$$\begin{pmatrix} p_{x'} \\ p_{t'} \end{pmatrix} = \Lambda \begin{pmatrix} p_x \\ p_t \end{pmatrix}$$

units: Λ only makes sense if same units for x, t, x', t'

Definition: $1 \text{ m} := (1/2.99792458 \times 10^8) \text{ s} \approx 10^{-8.5} \text{ s} \approx 3 \text{ ns}$

speed of light in a vacuum = $c = \frac{2.99792458 \times 10^8 \text{ m}}{1 \text{ s}}$

L = $\frac{1 \text{ s}}{1 \text{ s}} = 1$ (dimensionless)



SR: rapidity ϕ vs velocity β

What is ϕ ?





SR: rapidity ϕ vs velocity β

What is ϕ ?

observer A has worldline $(x, t) = (0, t)$





SR: rapidity ϕ vs velocity β

What is ϕ ?

observer B has worldline $(x', t') = (0, t')$





SR: rapidity ϕ vs velocity β

What is ϕ ?

observer B has worldline $(x', t') = (0, t')$

in A's coordinate system, B's worldline is:

$$\begin{pmatrix} 0 \\ t' \end{pmatrix} = \Lambda \begin{pmatrix} x \\ t \end{pmatrix}$$





SR: rapidity ϕ vs velocity β

What is ϕ ?

observer B has worldline $(x', t') = (0, t')$

in A's coordinate system, B's worldline is:

$$\Rightarrow \begin{pmatrix} x \\ t \end{pmatrix} = \Lambda^{-1} \begin{pmatrix} 0 \\ t' \end{pmatrix} = \begin{pmatrix} t' \sinh \phi \\ t' \cosh \phi \end{pmatrix}$$





SR: rapidity ϕ vs velocity β

What is ϕ ?

observer B has worldline $(x', t') = (0, t')$

in A's coordinate system, B's worldline is:

$$\Rightarrow \begin{pmatrix} x \\ t \end{pmatrix} = \Lambda^{-1} \begin{pmatrix} 0 \\ t' \end{pmatrix} = \begin{pmatrix} t' \sinh \phi \\ t' \cosh \phi \end{pmatrix}$$

$$\Rightarrow x = t' \sinh \phi$$





SR: rapidity ϕ vs velocity β

What is ϕ ?

observer B has worldline $(x', t') = (0, t')$

in A's coordinate system, B's worldline is:

$$\Rightarrow \begin{pmatrix} x \\ t \end{pmatrix} = \Lambda^{-1} \begin{pmatrix} 0 \\ t' \end{pmatrix} = \begin{pmatrix} t' \sinh \phi \\ t' \cosh \phi \end{pmatrix}$$

$$\Rightarrow x = t' \sinh \phi = t' \cosh \phi \frac{\sinh \phi}{\cosh \phi}$$





SR: rapidity ϕ vs velocity β

What is ϕ ?

observer B has worldline $(x', t') = (0, t')$

in A's coordinate system, B's worldline is:

$$\Rightarrow \begin{pmatrix} x \\ t \end{pmatrix} = \Lambda^{-1} \begin{pmatrix} 0 \\ t' \end{pmatrix} = \begin{pmatrix} t' \sinh \phi \\ t' \cosh \phi \end{pmatrix}$$

$$\Rightarrow x = t' \sinh \phi = t' \cosh \phi \tanh \phi$$





SR: rapidity ϕ vs velocity β

What is ϕ ?

observer B has worldline $(x', t') = (0, t')$

in A's coordinate system, B's worldline is:

$$\Rightarrow \begin{pmatrix} x \\ t \end{pmatrix} = \Lambda^{-1} \begin{pmatrix} 0 \\ t' \end{pmatrix} = \begin{pmatrix} t' \sinh \phi \\ t' \cosh \phi \end{pmatrix}$$

$$\Rightarrow x = t' \sinh \phi = t \tanh \phi$$





SR: rapidity ϕ vs velocity β

What is ϕ ?

observer B has worldline $(x', t') = (0, t')$

in A's coordinate system, B's worldline is:

$$\Rightarrow \begin{pmatrix} x \\ t \end{pmatrix} = \Lambda^{-1} \begin{pmatrix} 0 \\ t' \end{pmatrix} = \begin{pmatrix} t' \sinh \phi \\ t' \cosh \phi \end{pmatrix}$$

$$\Rightarrow x = t' \sinh \phi = t \tanh \phi = \beta t$$

where velocity $\beta := v/c \equiv v = \tanh \phi$





SR: calibration

Where does $(x', t') = (0, 1)$ lie for observer A?





SR: calibration

Where does $(x', t') = (0, 1)$ lie for observer A?

Where does $(x', t') = (1, 0)$ lie for observer A?

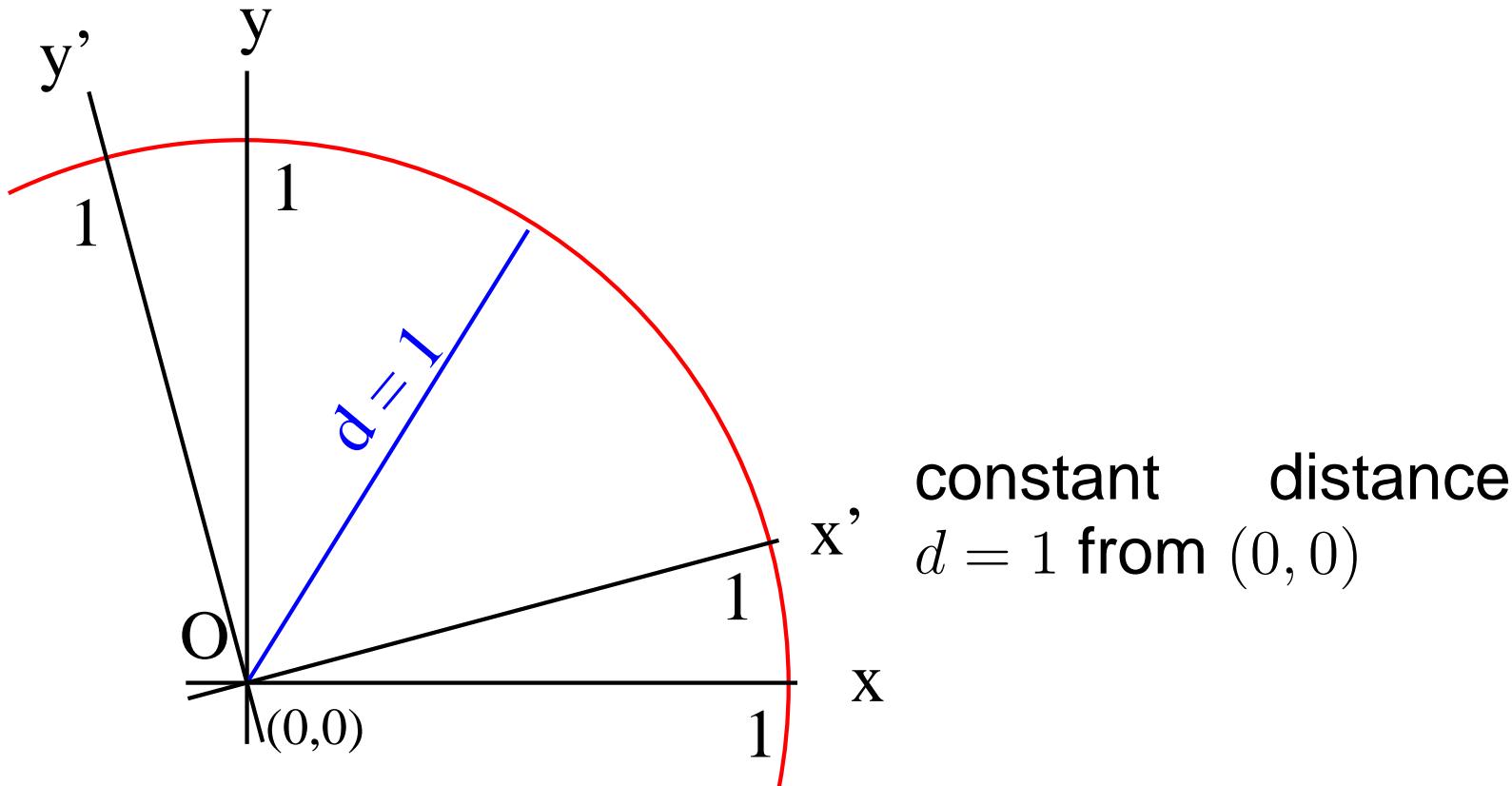




SR: calibration

Where does $(x', t') = (0, 1)$ lie for observer A?

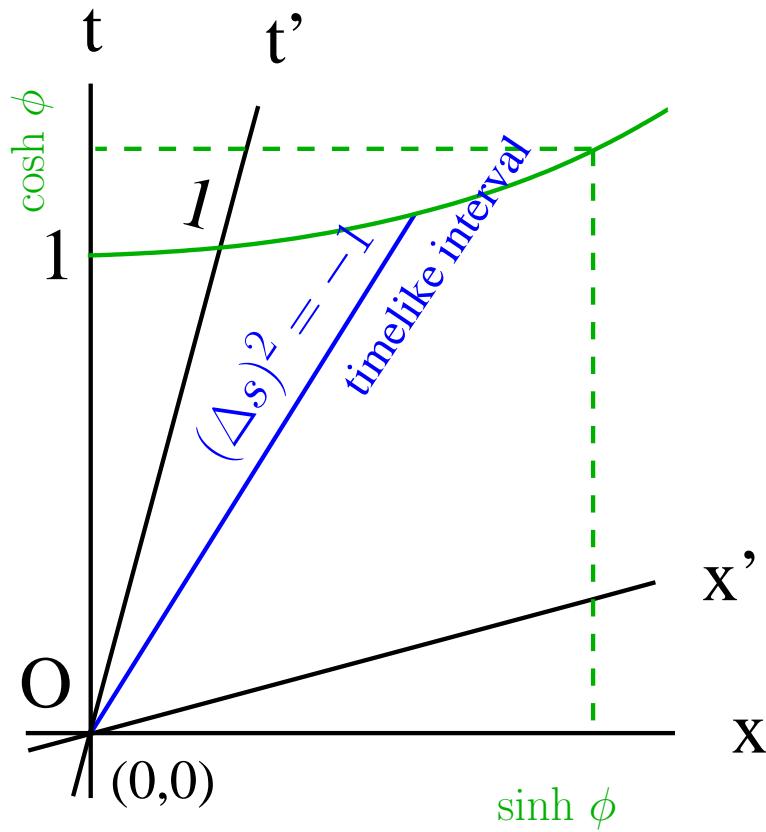
Where does $(x', t') = (1, 0)$ lie for observer A?



SR: calibration

Where does $(x', t') = (0, 1)$ lie for observer A?

Where does $(x', t') = (1, 0)$ lie for observer A?



constant interval

$(\Delta s)^2 = -1$ from $(0, 0)$

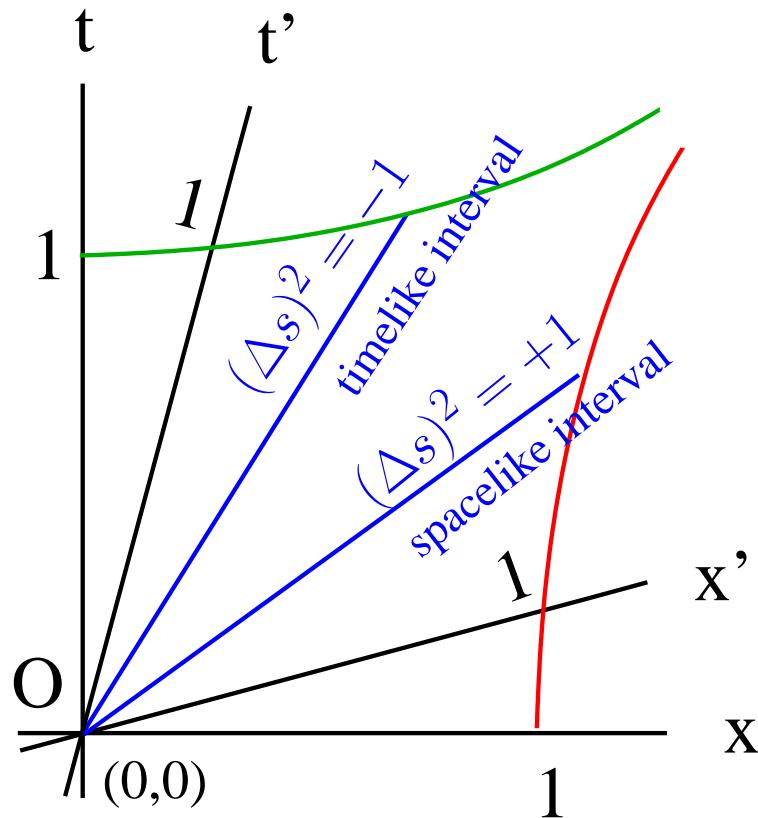
$$\Lambda^{-1} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} \sinh \phi \\ \cosh \phi \end{pmatrix}$$



SR: calibration

Where does $(x', t') = (0, 1)$ lie for observer A?

Where does $(x', t') = (1, 0)$ lie for observer A?



constant interval

$(\Delta s)^2 = +1$ from $(0, 0)$

$$\Lambda^{-1} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} \cosh \phi \\ \sinh \phi \end{pmatrix}$$





SR: calibration

Where does $(x', t') = (0, 1)$ lie for observer A?

Where does $(x', t') = (1, 0)$ lie for observer A?

Can high ϕ push the t' axis close to the x axis?



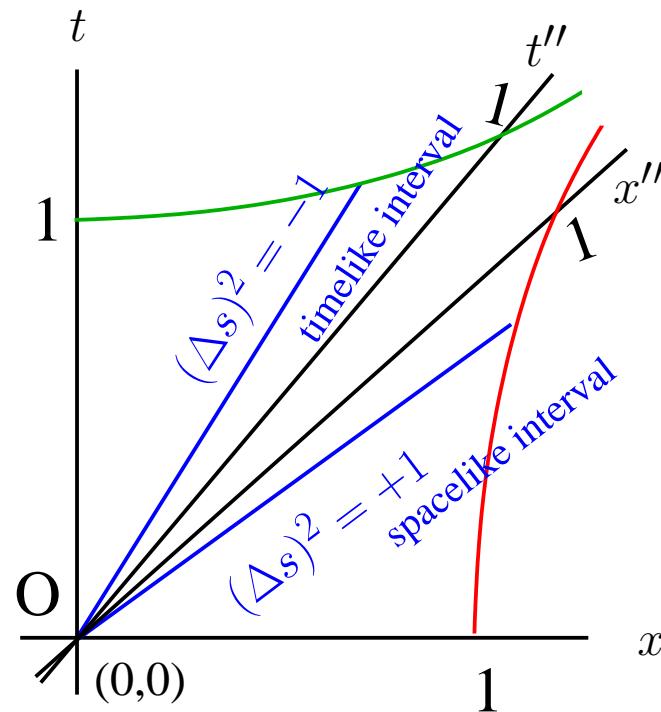


SR: calibration

Where does $(x', t') = (0, 1)$ lie for observer A?

Where does $(x', t') = (1, 0)$ lie for observer A?

Can high ϕ push the t' axis close to the x axis?



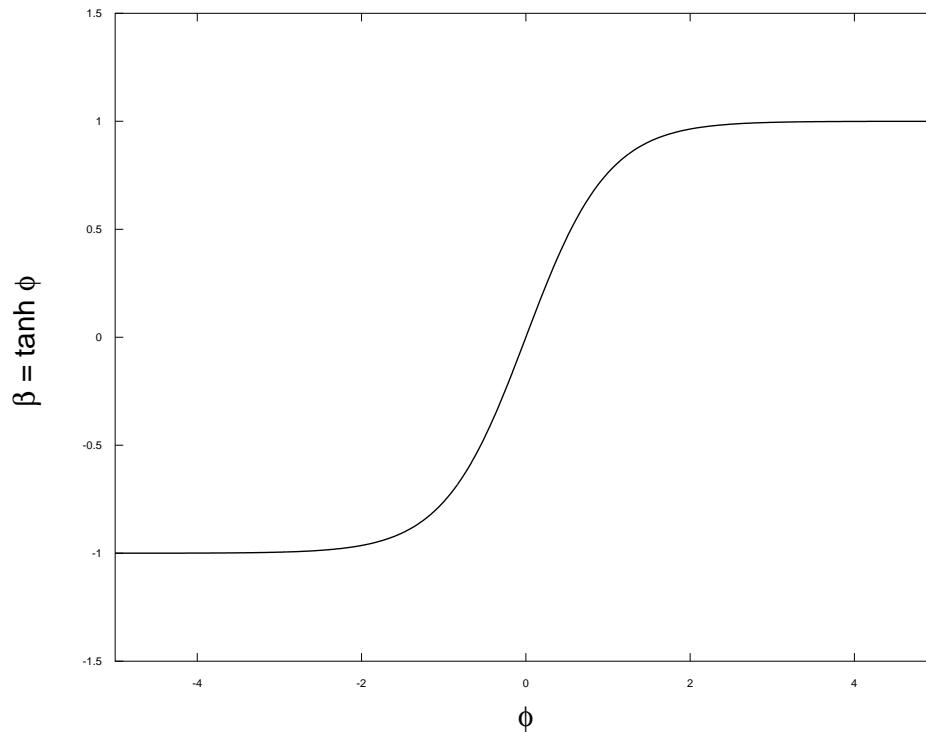


SR: calibration

Where does $(x', t') = (0, 1)$ lie for observer A?

Where does $(x', t') = (1, 0)$ lie for observer A?

Can high ϕ push the t' axis close to the x axis?



SR: effect of Λ on $x = t$ (photons)

What happens to a photon under Lorentz transformation?



SR: effect of Λ on $x = t$ (photons)

What happens to a photon under Lorentz transformation?



SR: effect of Λ on $x = t$ (photons)

What happens to a photon under Lorentz transformation?

Observer A: photon spacetime path is the set of spacetime events $\{(t, t) \mid t_1 < t < t_2\}$ for some t_1, t_2





SR: effect of Λ on $x = t$ (photons)

What happens to a photon under Lorentz transformation?

Observer A: photon spacetime path is the set of spacetime events $\{(t, t) \mid t_1 < t < t_2\}$ for some t_1, t_2

$$\begin{pmatrix} x' \\ t' \end{pmatrix} = \begin{pmatrix} \cosh \phi & -\sinh \phi \\ -\sinh \phi & \cosh \phi \end{pmatrix} \begin{pmatrix} t \\ t \end{pmatrix}$$



SR: effect of Λ on $x = t$ (photons)



What happens to a photon under Lorentz transformation?

Observer A: photon spacetime path is the set of spacetime events $\{(t, t) \mid t_1 < t < t_2\}$ for some t_1, t_2

$$\begin{pmatrix} x' \\ t' \end{pmatrix} = \begin{pmatrix} \cosh \phi & -\sinh \phi \\ -\sinh \phi & \cosh \phi \end{pmatrix} \begin{pmatrix} t \\ t \end{pmatrix}$$
$$= \begin{pmatrix} (\cosh \phi - \sinh \phi)t \\ (-\sinh \phi + \cosh \phi)t \end{pmatrix}$$



SR: effect of Λ on $x = t$ (photons)



What happens to a photon under Lorentz transformation?

Observer A: photon spacetime path is the set of spacetime events $\{(t, t) \mid t_1 < t < t_2\}$ for some t_1, t_2

$$\begin{pmatrix} x' \\ t' \end{pmatrix} = \begin{pmatrix} \cosh \phi & -\sinh \phi \\ -\sinh \phi & \cosh \phi \end{pmatrix} \begin{pmatrix} t \\ t \end{pmatrix}$$
$$= \begin{pmatrix} (\cosh \phi - \sinh \phi)t \\ (-\sinh \phi + \cosh \phi)t \end{pmatrix}$$

\Rightarrow path is $x' = (\cosh \phi - \sinh \phi)t = t'$ i.e. the set of spacetime events

$$\{(t', t') \mid (\cosh \phi - \sinh \phi)t_1 < t' < (\cosh \phi - \sinh \phi)t_2\}$$



SR: effect of Λ on $x = t$ (photons)



What happens to a photon under Lorentz transformation?

Observer A: photon spacetime path is the set of spacetime events $\{(t, t) \mid t_1 < t < t_2\}$ for some t_1, t_2

$$\begin{pmatrix} x' \\ t' \end{pmatrix} = \begin{pmatrix} \cosh \phi & -\sinh \phi \\ -\sinh \phi & \cosh \phi \end{pmatrix} \begin{pmatrix} t \\ t \end{pmatrix}$$
$$= \begin{pmatrix} (\cosh \phi - \sinh \phi)t \\ (-\sinh \phi + \cosh \phi)t \end{pmatrix}$$

\Rightarrow path is $x' = (\cosh \phi - \sinh \phi)t = t'$ i.e. the set of spacetime events

$$\{(t', t') \mid (\cosh \phi - \sinh \phi)t_1 < t' < (\cosh \phi - \sinh \phi)t_2\}$$

- photon speed same in both reference frames



SR: effect of Λ on $x = t$ (photons)



What happens to a photon under Lorentz transformation?

Observer A: photon spacetime path is the set of spacetime events $\{(t, t) \mid t_1 < t < t_2\}$ for some t_1, t_2

$$\begin{pmatrix} x' \\ t' \end{pmatrix} = \begin{pmatrix} \cosh \phi & -\sinh \phi \\ -\sinh \phi & \cosh \phi \end{pmatrix} \begin{pmatrix} t \\ t \end{pmatrix}$$
$$= \begin{pmatrix} (\cosh \phi - \sinh \phi)t \\ (-\sinh \phi + \cosh \phi)t \end{pmatrix}$$

\Rightarrow path is $x' = (\cosh \phi - \sinh \phi)t = t'$ i.e. the set of spacetime events

$$\{(t', t') \mid (\cosh \phi - \sinh \phi)t_1 < t' < (\cosh \phi - \sinh \phi)t_2\}$$

- photon speed same in both reference frames

- w:Michelson-Morley experiment (1887)





SR: adding velocities

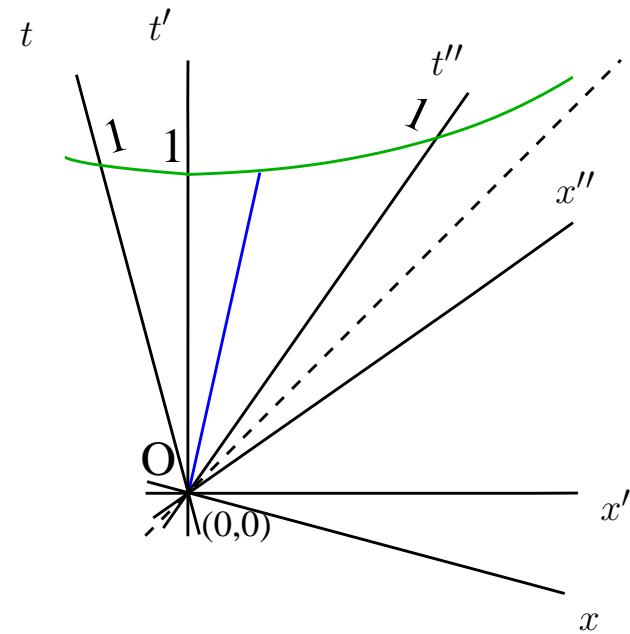
interstellar ark travels at $\beta_1 = 0.1$ from Sun, sends out rocket at $\beta_2 = 0.5$; rocket's speed β_3 in Sun frame =?





SR: adding velocities

interstellar ark travels at $\beta_1 = 0.1$ from Sun, sends out rocket at $\beta_2 = 0.5$; rocket's speed β_3 in Sun frame =?



$$\begin{pmatrix} x' \\ t' \end{pmatrix} = \begin{pmatrix} \cosh \phi_1 & -\sinh \phi_1 \\ -\sinh \phi_1 & \cosh \phi_1 \end{pmatrix} \begin{pmatrix} x \\ t \end{pmatrix}$$

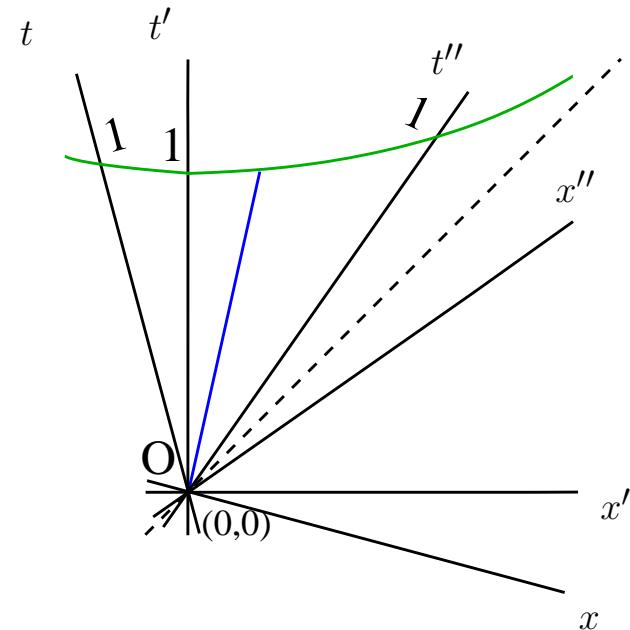
where $\tanh \phi_1 = \beta_1 = 0.1$





SR: adding velocities

interstellar ark travels at $\beta_1 = 0.1$ from Sun, sends out rocket at $\beta_2 = 0.5$; rocket's speed β_3 in Sun frame =?



$$\begin{pmatrix} x'' \\ t'' \end{pmatrix} = \begin{pmatrix} \cosh \phi_2 & -\sinh \phi_2 \\ -\sinh \phi_2 & \cosh \phi_2 \end{pmatrix} \begin{pmatrix} x' \\ t' \end{pmatrix}$$

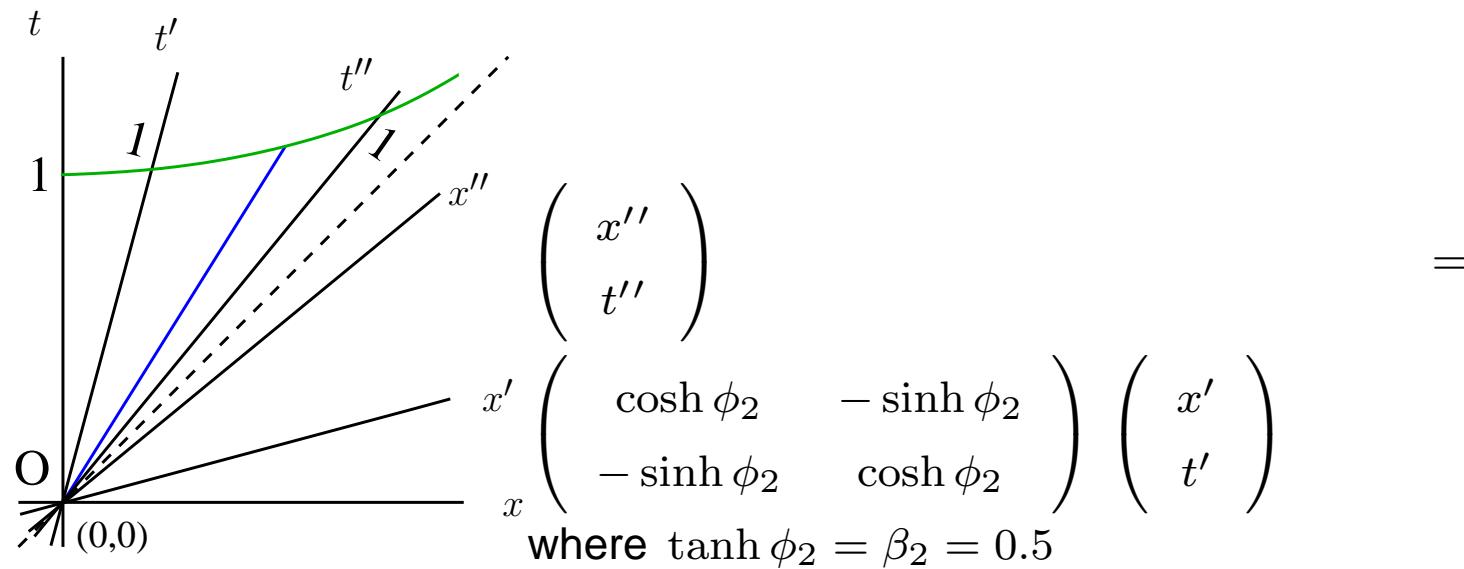
where $\tanh \phi_2 = \beta_2 = 0.5$





SR: adding velocities

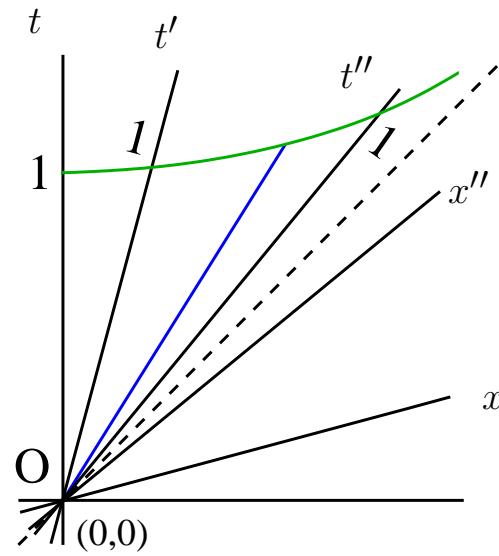
interstellar ark travels at $\beta_1 = 0.1$ from Sun, sends out rocket at $\beta_2 = 0.5$; rocket's speed β_3 in Sun frame =?





SR: adding velocities

interstellar ark travels at $\beta_1 = 0.1$ from Sun, sends out rocket at $\beta_2 = 0.5$; rocket's speed β_3 in Sun frame =?



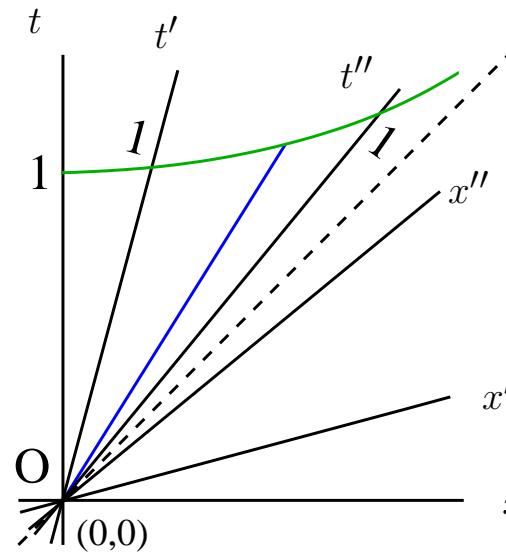
$$x \begin{pmatrix} x'' \\ t'' \end{pmatrix} = \Lambda(\phi_2)\Lambda(\phi_1) \begin{pmatrix} x \\ t \end{pmatrix}$$





SR: adding velocities

interstellar ark travels at $\beta_1 = 0.1$ from Sun, sends out rocket at $\beta_2 = 0.5$; rocket's speed β_3 in Sun frame =?



$$x' \begin{pmatrix} x'' \\ t'' \end{pmatrix} = \Lambda(\phi_2)\Lambda(\phi_1) \begin{pmatrix} x \\ t \end{pmatrix}$$

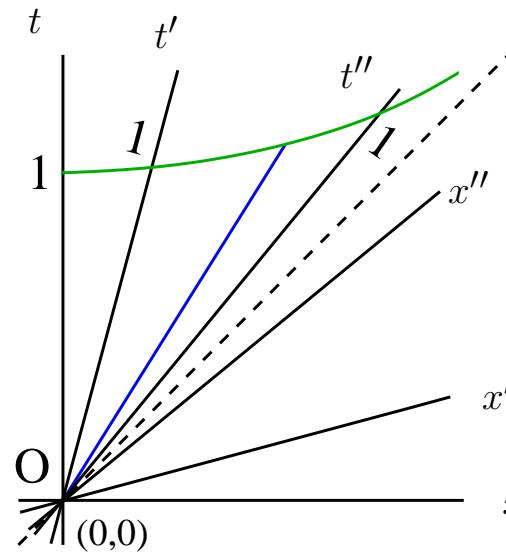
but $\Lambda(\phi_2)\Lambda(\phi_1) = \Lambda(\phi_1 + \phi_2)$





SR: adding velocities

interstellar ark travels at $\beta_1 = 0.1$ from Sun, sends out rocket at $\beta_2 = 0.5$; rocket's speed β_3 in Sun frame =?



$$x' \begin{pmatrix} x'' \\ t'' \end{pmatrix} = \Lambda(\phi_2)\Lambda(\phi_1) \begin{pmatrix} x \\ t \end{pmatrix}$$

but $\Lambda(\phi_2)\Lambda(\phi_1) = \Lambda(\phi_1 + \phi_2)$

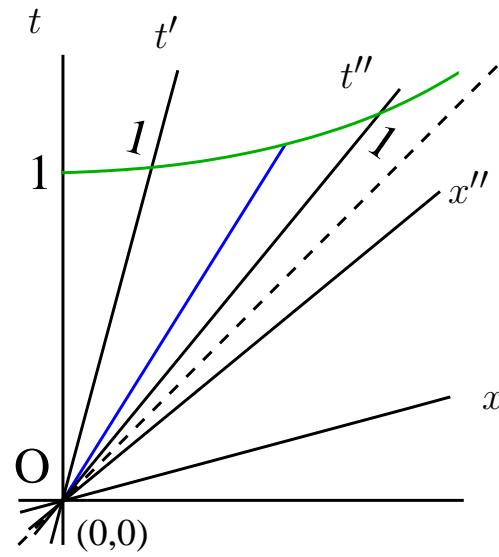
cf. rotation θ_1 "plus" rotation θ_2 = rotation $\theta_1 + \theta_2$





SR: adding velocities

interstellar ark travels at $\beta_1 = 0.1$ from Sun, sends out rocket at $\beta_2 = 0.5$; rocket's speed β_3 in Sun frame =?



$$x' \begin{pmatrix} x'' \\ t'' \end{pmatrix} = \Lambda(\phi_2)\Lambda(\phi_1) \begin{pmatrix} x \\ t \end{pmatrix}$$

but $\Lambda(\phi_2)\Lambda(\phi_1) = \Lambda(\phi_1 + \phi_2)$

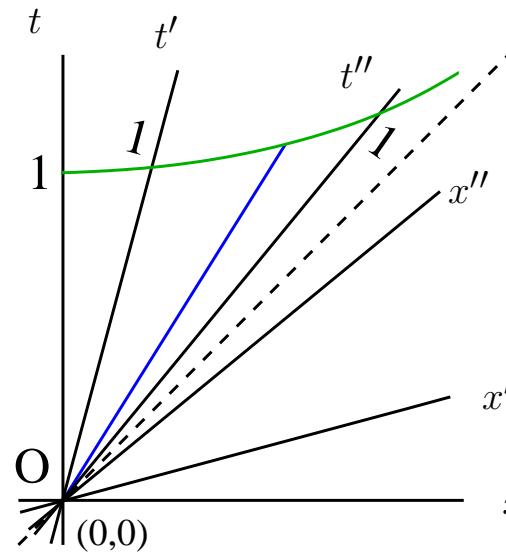
so $\beta_3 = \tanh(\phi_1 + \phi_2)$





SR: adding velocities

interstellar ark travels at $\beta_1 = 0.1$ from Sun, sends out rocket at $\beta_2 = 0.5$; rocket's speed β_3 in Sun frame =?



$$x' \begin{pmatrix} x'' \\ t'' \end{pmatrix} = \Lambda(\phi_2)\Lambda(\phi_1) \begin{pmatrix} x \\ t \end{pmatrix}$$

but $\Lambda(\phi_2)\Lambda(\phi_1) = \Lambda(\phi_1 + \phi_2)$

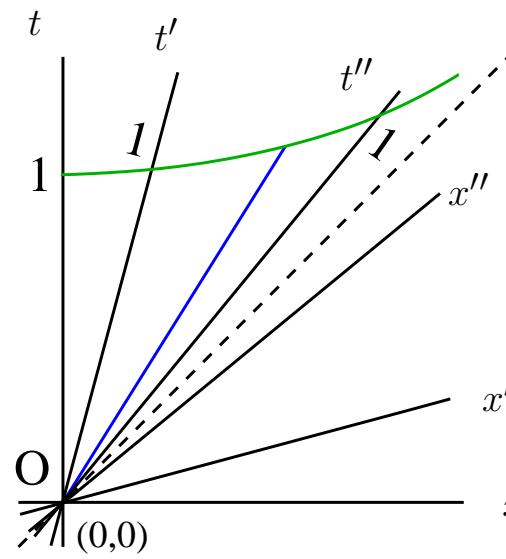
$$\text{so } \beta_3 = \tanh(\phi_1 + \phi_2) = \frac{\tanh \phi_1 + \tanh \phi_2}{1 + \tanh \phi_1 \tanh \phi_2}$$





SR: adding velocities

interstellar ark travels at $\beta_1 = 0.1$ from Sun, sends out rocket at $\beta_2 = 0.5$; rocket's speed β_3 in Sun frame =?



$$x \begin{pmatrix} x'' \\ t'' \end{pmatrix} = \Lambda(\phi_2)\Lambda(\phi_1) \begin{pmatrix} x \\ t \end{pmatrix}$$

but $\Lambda(\phi_2)\Lambda(\phi_1) = \Lambda(\phi_1 + \phi_2)$

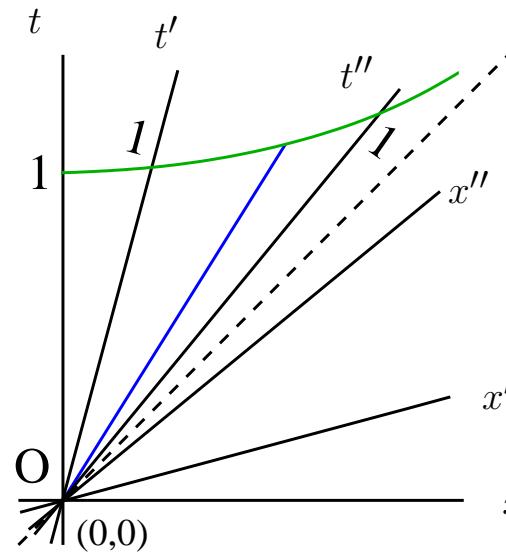
$$\text{so } \beta_3 = \tanh(\phi_1 + \phi_2) = \frac{\beta_1 + \beta_2}{1 + \beta_1 \beta_2}$$





SR: adding velocities

interstellar ark travels at $\beta_1 = 0.1$ from Sun, sends out rocket at $\beta_2 = 0.5$; rocket's speed β_3 in Sun frame =?



$$x \begin{pmatrix} x'' \\ t'' \end{pmatrix} = \Lambda(\phi_2)\Lambda(\phi_1) \begin{pmatrix} x \\ t \end{pmatrix}$$

but $\Lambda(\phi_2)\Lambda(\phi_1) = \Lambda(\phi_1 + \phi_2)$

$$\text{so } \beta_3 = \tanh(\phi_1 + \phi_2) = \frac{\beta_1 + \beta_2}{1 + \beta_1 \beta_2} = \frac{0.1 + 0.5}{1 + 0.1 \times 0.5} \approx 0.57$$





SR: Lorentz factor

Λ : alternative to hyperbolic trig functions





SR: Lorentz factor

Λ : alternative to hyperbolic trig functions

$$\Lambda(\phi) := \begin{pmatrix} \cosh \phi & -\sinh \phi \\ -\sinh \phi & \cosh \phi \end{pmatrix}$$

$$\cosh \phi := \frac{e^\phi + e^{-\phi}}{2}$$

$$\sinh \phi := \frac{e^\phi - e^{-\phi}}{2}$$

w:**hyperbolic function**





SR: Lorentz factor

Λ : alternative to hyperbolic trig functions

$$\Lambda(\beta) := \begin{pmatrix} \gamma & -\beta\gamma \\ -\beta\gamma & \gamma \end{pmatrix}$$

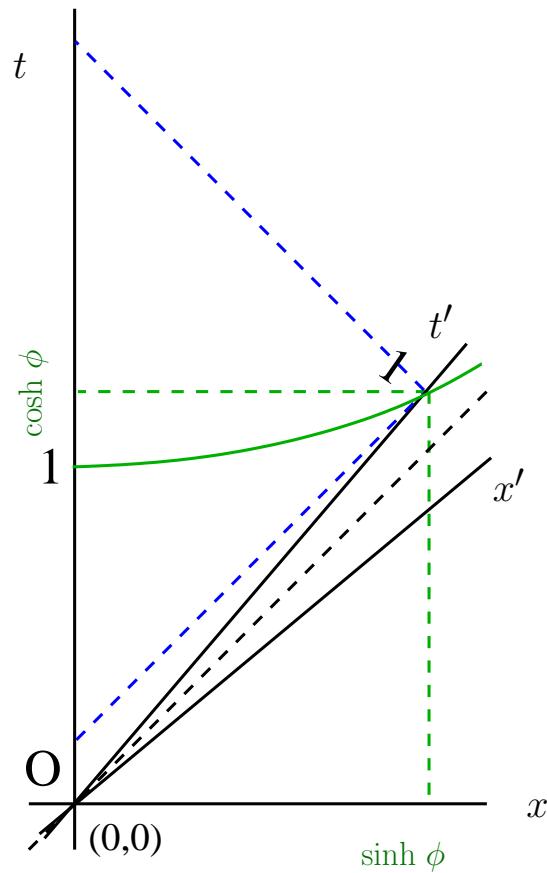
$$\begin{aligned}\beta &= \tanh \phi \\ \gamma &:= (1 - \beta^2)^{-1/2} = \\ &\text{Lorentz factor}\end{aligned}$$

$$\begin{aligned}\gamma &= \cosh \phi \\ \beta\gamma &= \sinh \phi\end{aligned}$$



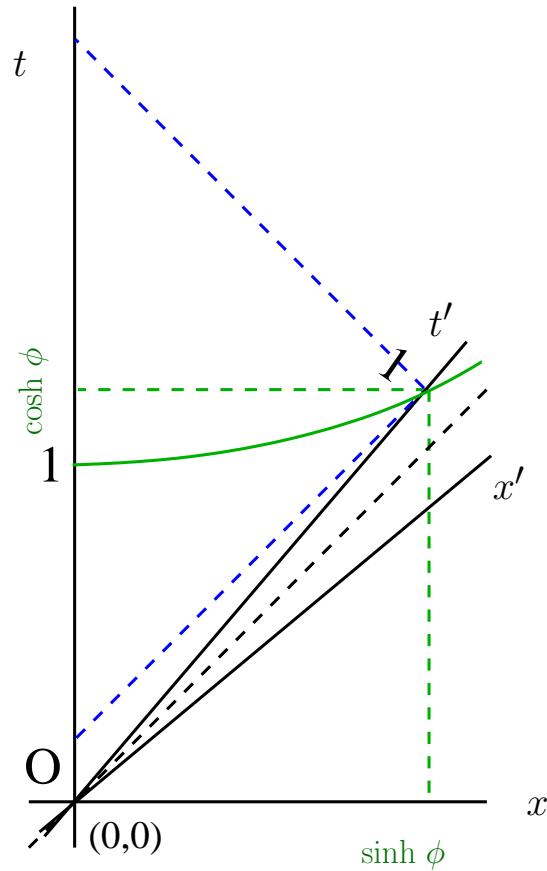


SR: worldline time dilation





SR: worldline time dilation

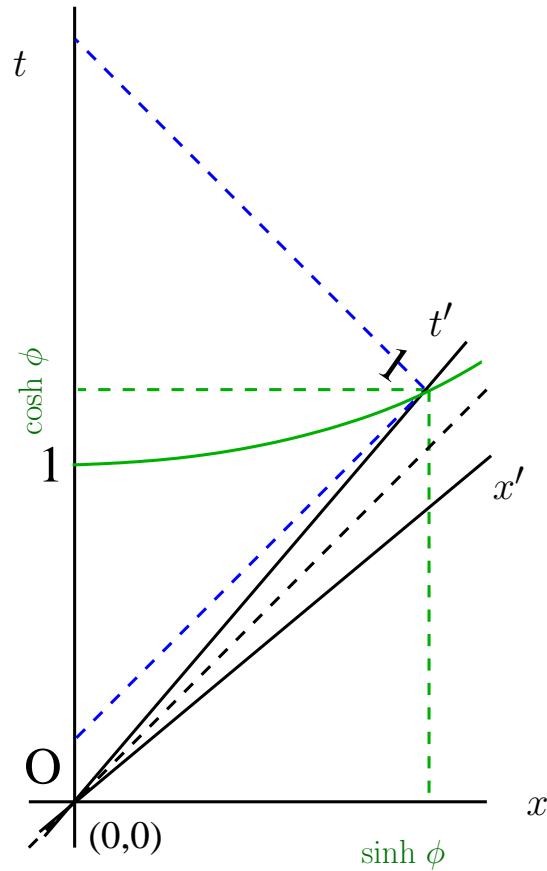


$$\cosh \phi \equiv \gamma$$





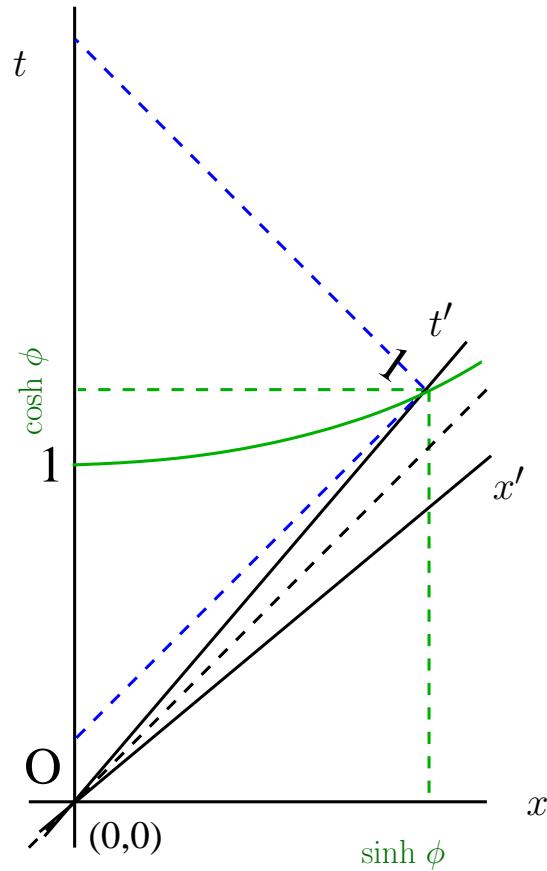
SR: worldline time dilation



$$\cosh \phi \equiv \gamma \equiv \frac{1}{\sqrt{1-\beta^2}}$$



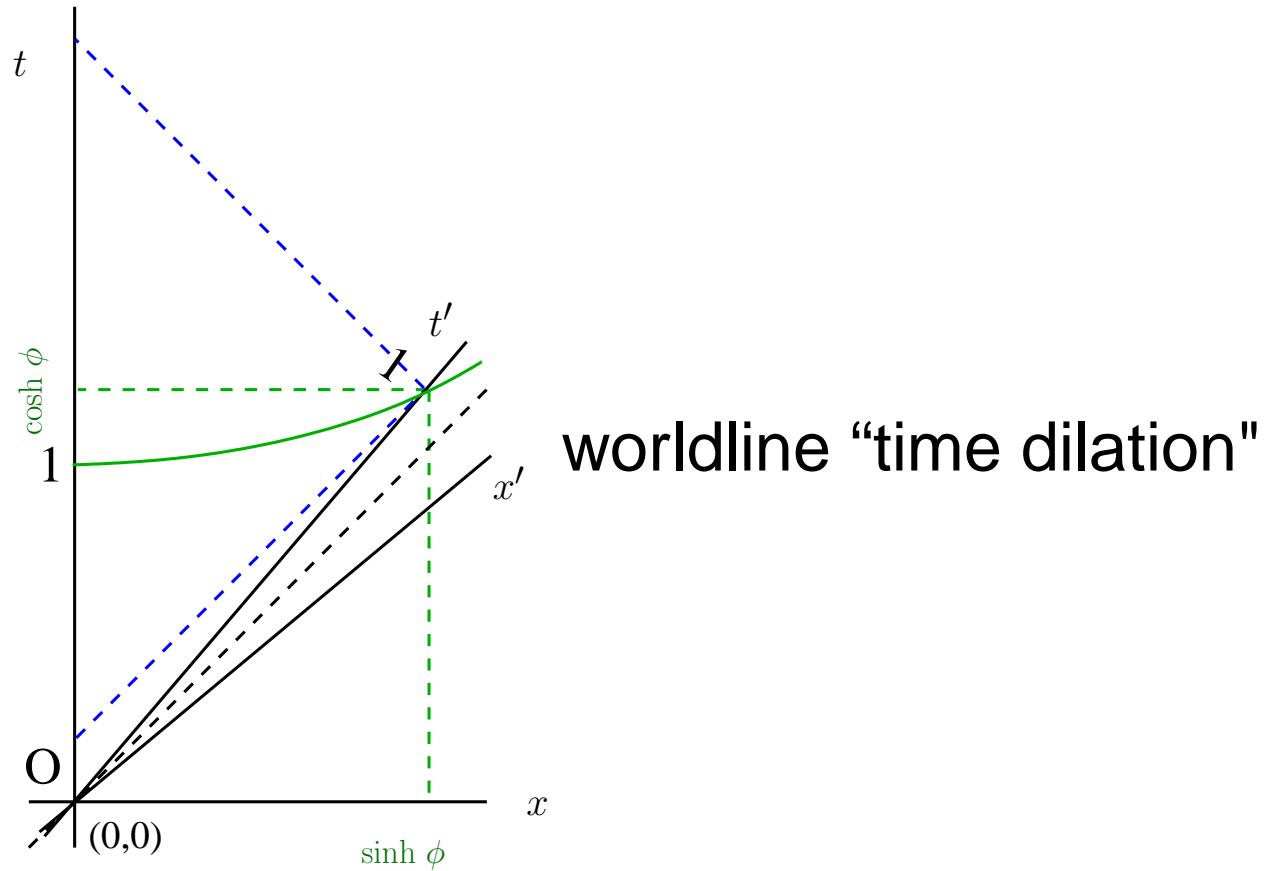
SR: worldline time dilation



$$\cosh \phi \equiv \gamma \equiv \frac{1}{\sqrt{1-\beta^2}} > 1$$



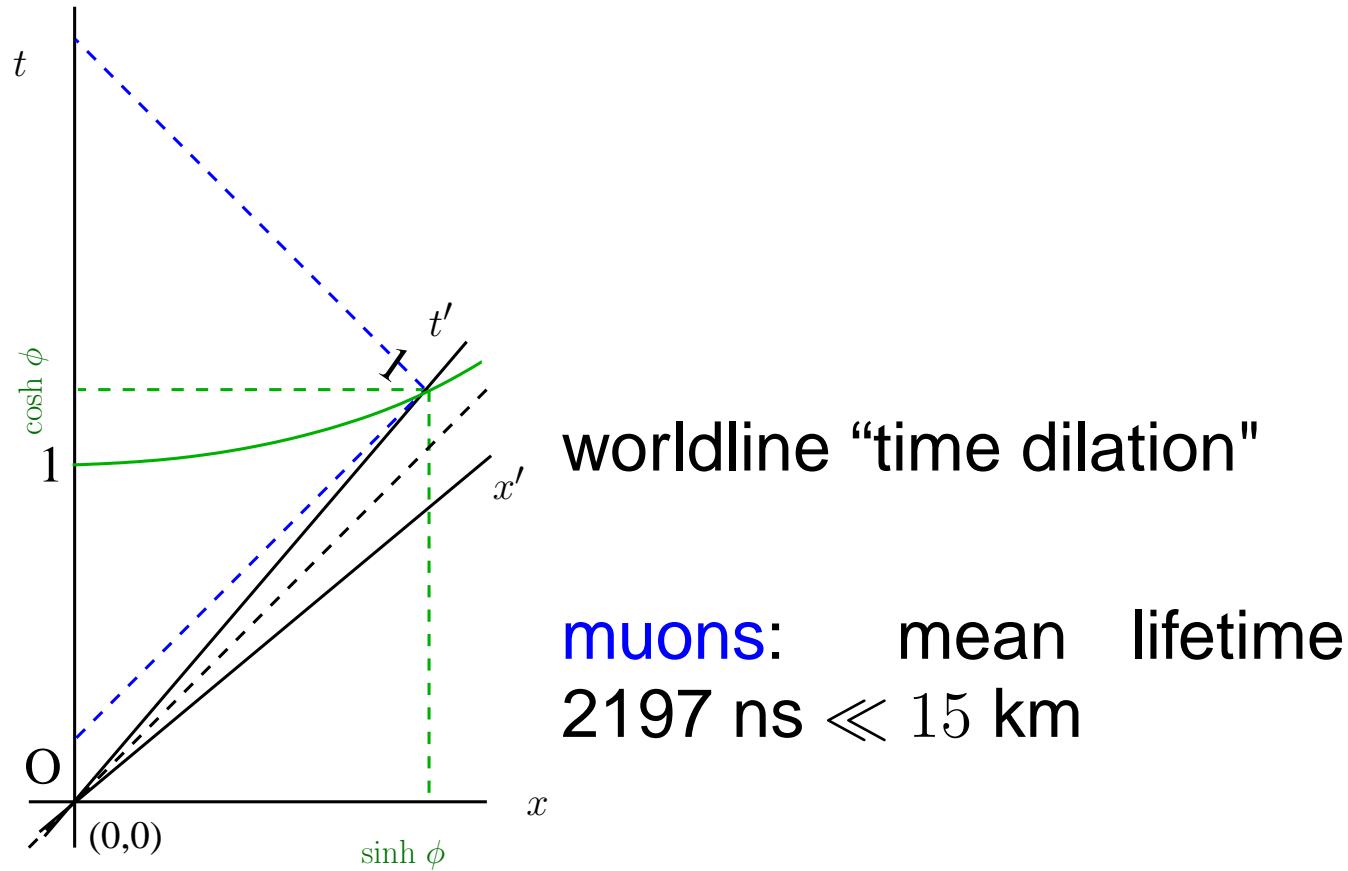
SR: worldline time dilation



$$\cosh \phi \equiv \gamma \equiv \frac{1}{\sqrt{1-\beta^2}} > 1$$

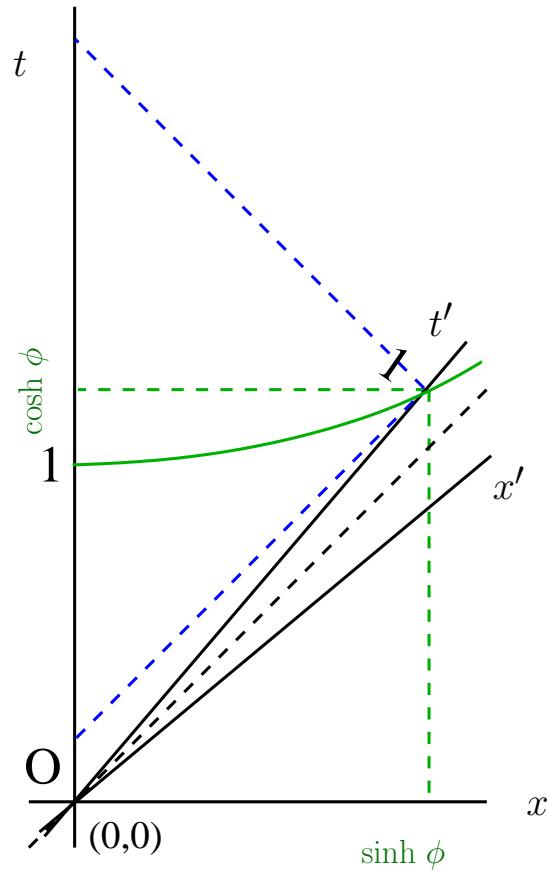


SR: worldline time dilation



$$\cosh \phi \equiv \gamma \equiv \frac{1}{\sqrt{1-\beta^2}} > 1$$

SR: worldline time dilation



worldline “time dilation”

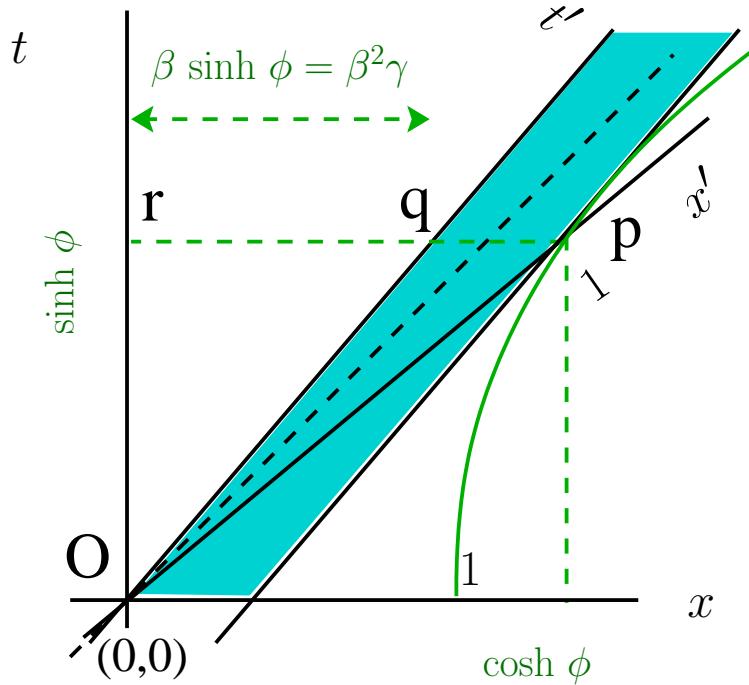
muons: mean lifetime
 $2197 \text{ ns} \ll 15 \text{ km}$

time dilation \Rightarrow muons
 can hit the ground

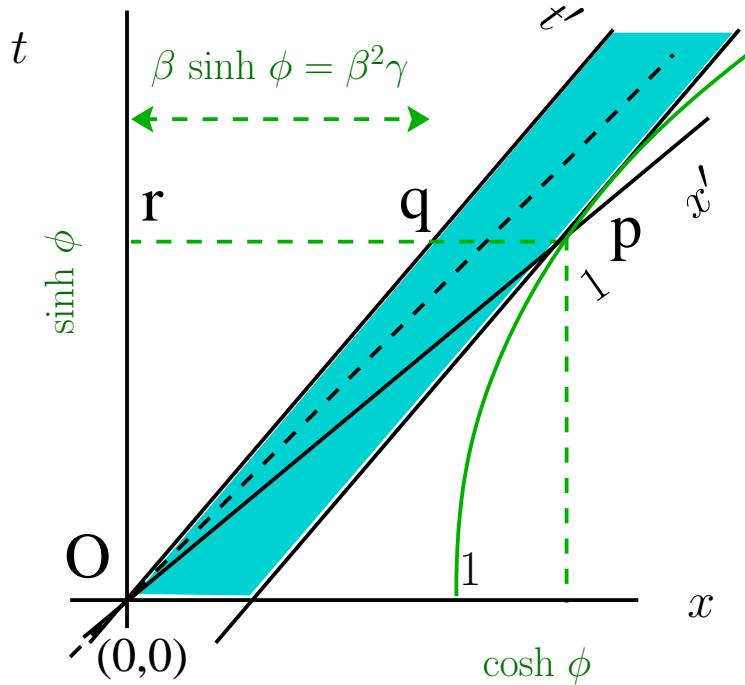
$$\cosh \phi \equiv \gamma \equiv \frac{1}{\sqrt{1-\beta^2}} > 1$$



SR: worldsheet space contraction



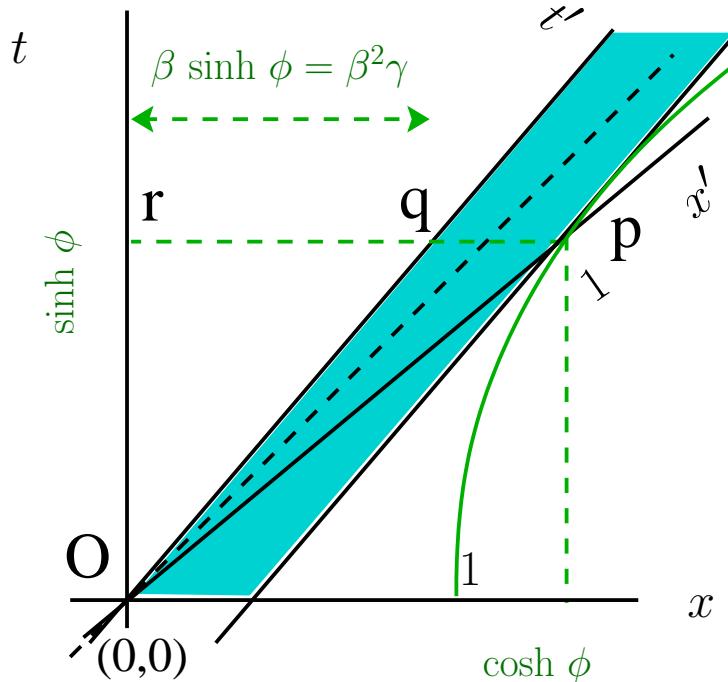
SR: worldsheet space contraction



$$\sqrt{\Delta s^2(q, p)} =$$

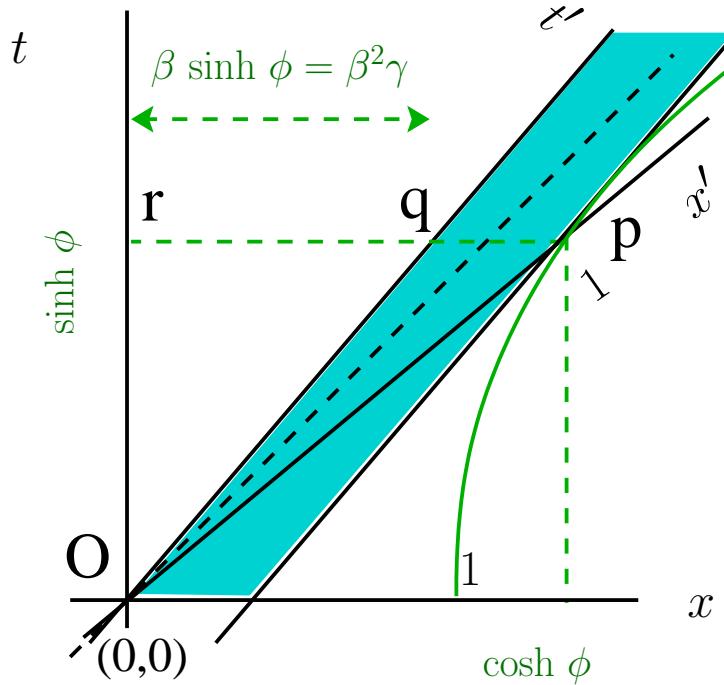


SR: worldsheet space contraction



$$\sqrt{\Delta s^2(q, p)} = \cosh \phi - \beta \sinh \phi$$

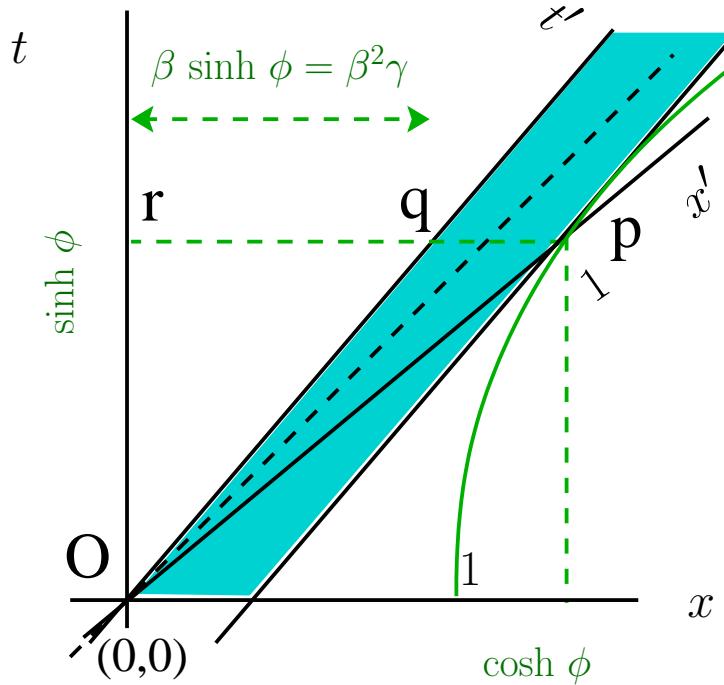
SR: worldsheet space contraction



$$\sqrt{\Delta s^2(q, p)} = \gamma - \beta\beta\gamma$$

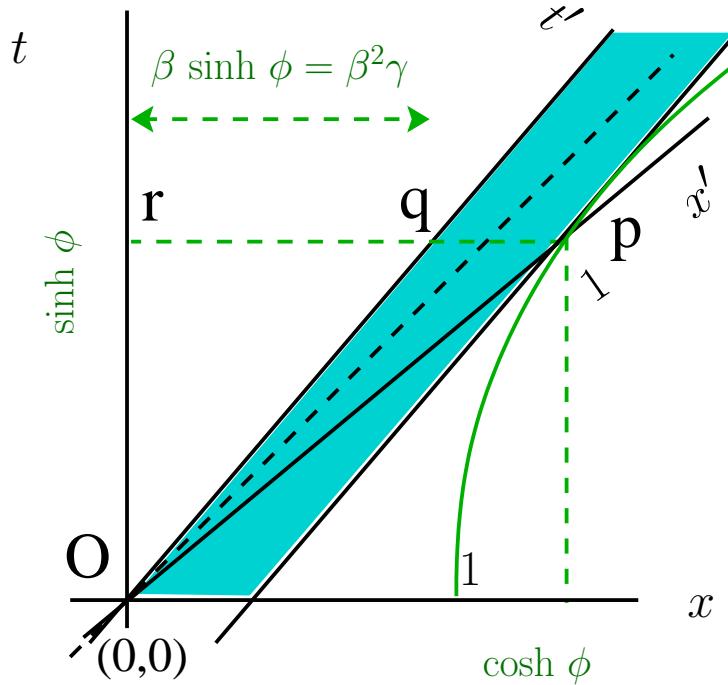


SR: worldsheet space contraction



$$\sqrt{\Delta s^2(q, p)} = (1 - \beta^2)\gamma$$

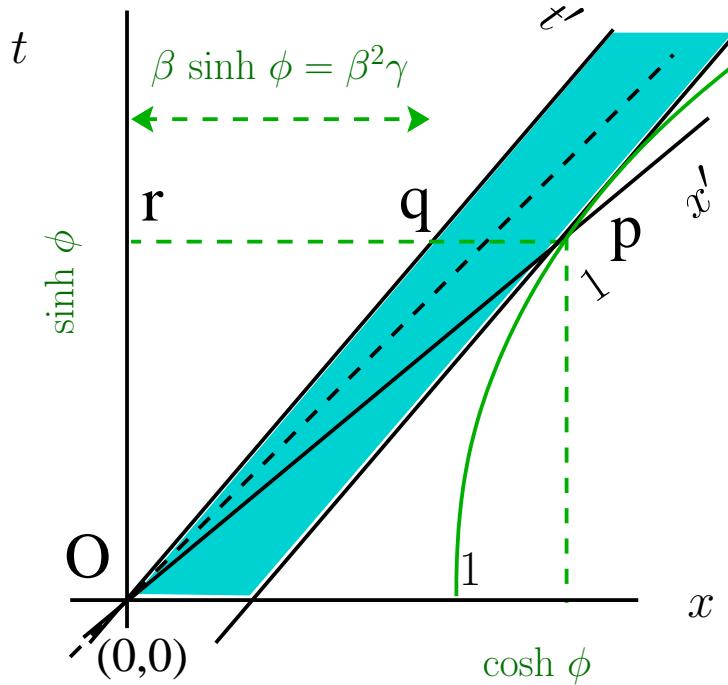
SR: worldsheet space contraction



$$\sqrt{\Delta s^2(q, p)} = (1 - \beta^2)^{1+(-1/2)}$$

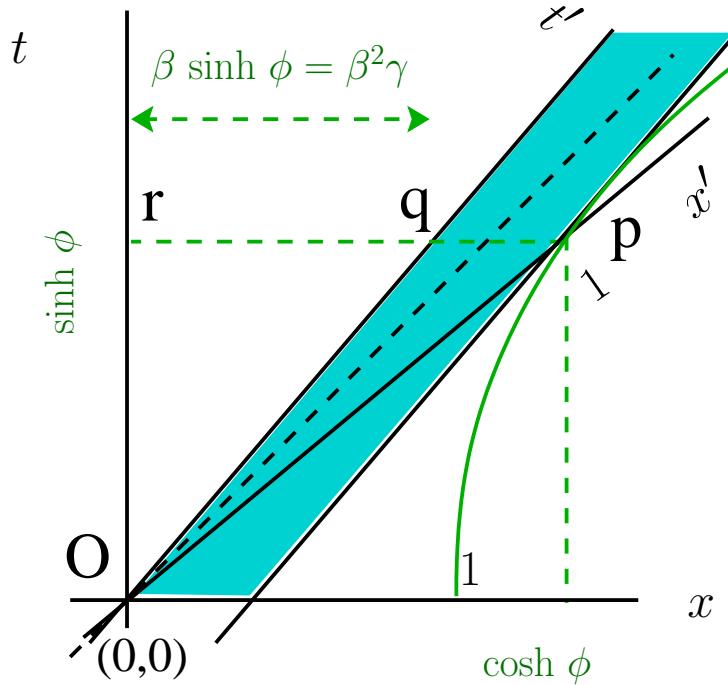


SR: worldsheet space contraction



$$\sqrt{\Delta s^2(q, p)} = (1 - \beta^2)^{1/2}$$

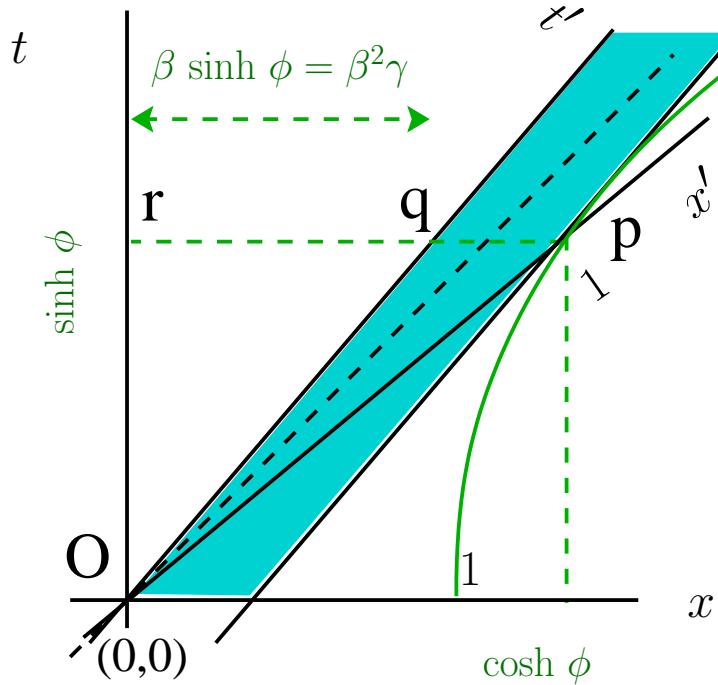
SR: worldsheet space contraction



$$\sqrt{\Delta s^2(q, p)} = \gamma^{-1} < 1$$

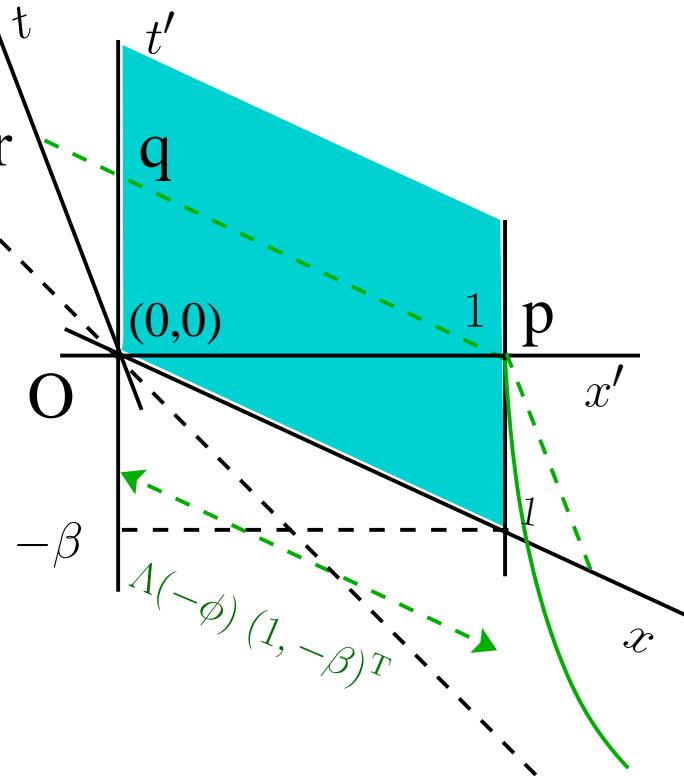


SR: worldsheet space contraction

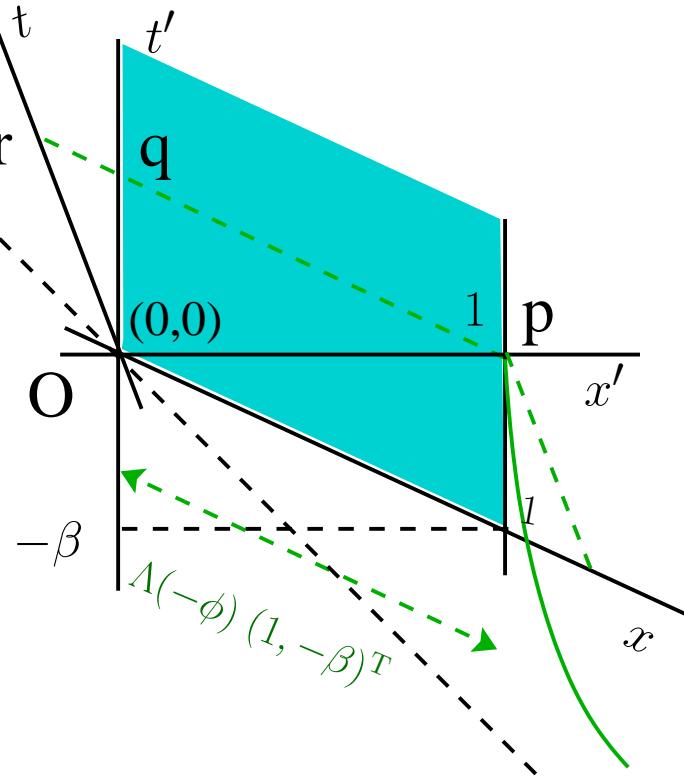


$$\sqrt{\Delta s^2(q, p)} = \gamma^{-1} < 1 \quad \text{worldsheet "space contraction"}$$

SR: worldsheet space contraction

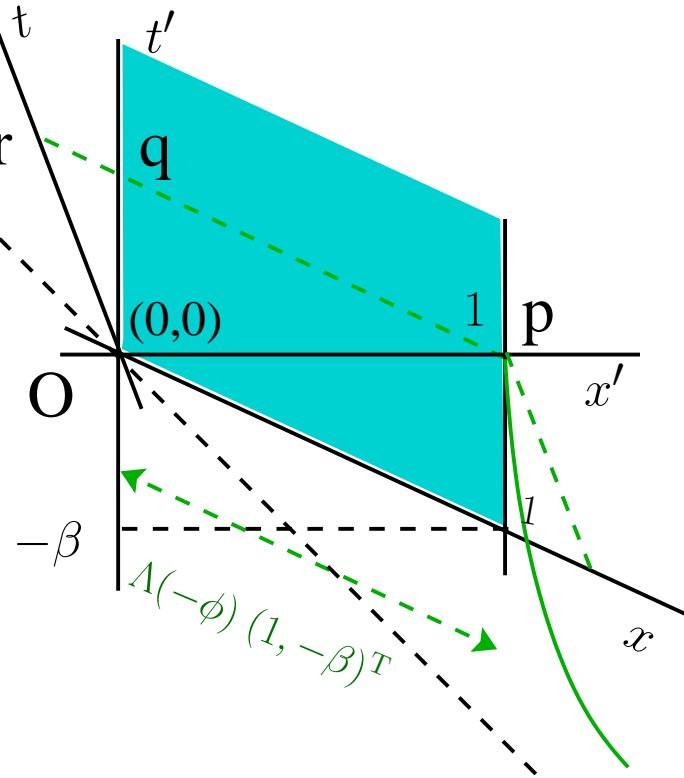


SR: worldsheet space contraction



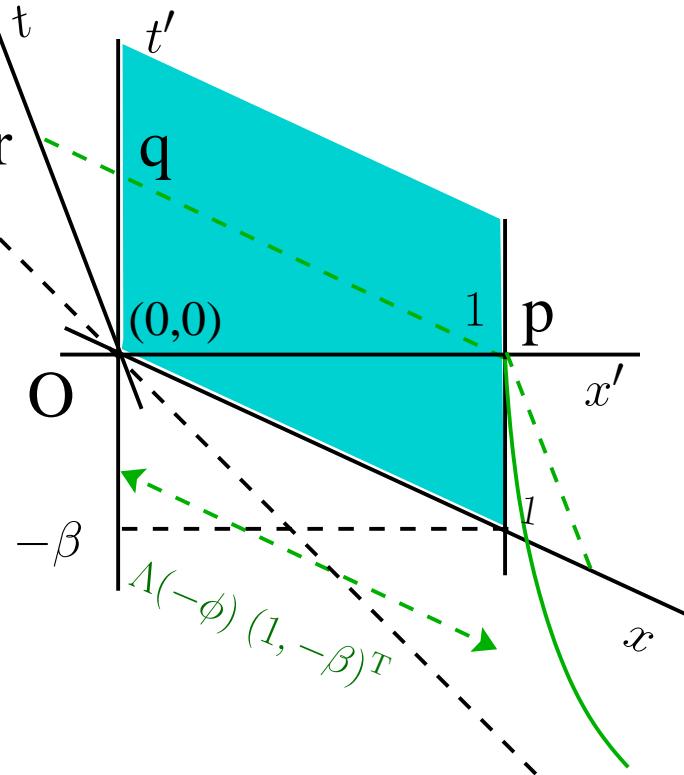
$$\Lambda^{-1} \begin{pmatrix} 1 \\ -\beta \end{pmatrix} = \begin{pmatrix} \cosh \phi - \beta \sinh \phi \\ \sinh \phi - \beta \cosh \phi \end{pmatrix}$$

SR: worldsheet space contraction



$$\Lambda^{-1} \begin{pmatrix} 1 \\ -\beta \end{pmatrix} = \begin{pmatrix} \cosh \phi (1 - \beta^2) \\ \cosh \phi (\tanh \phi - \tanh \phi) \end{pmatrix}$$

SR: worldsheet space contraction

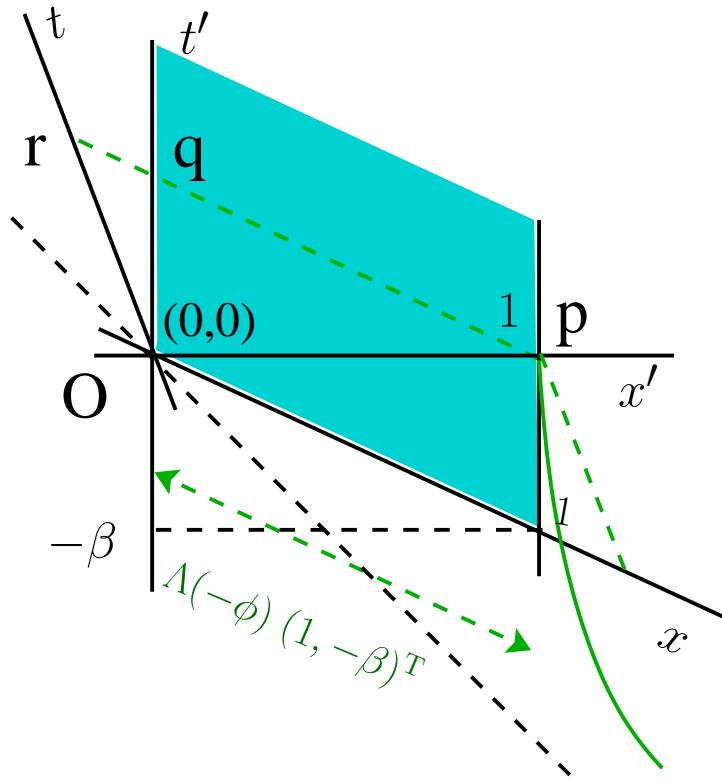


$$\Lambda^{-1} \begin{pmatrix} 1 \\ -\beta \end{pmatrix} = \begin{pmatrix} \gamma(1 - \beta^2) \\ 0 \end{pmatrix}$$





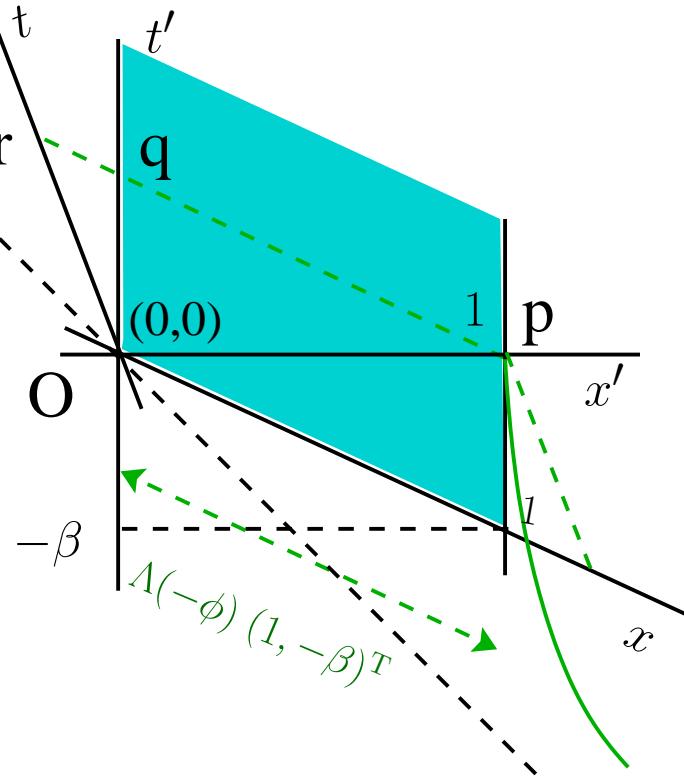
SR: worldsheet space contraction



$$\Lambda^{-1} \begin{pmatrix} 1 \\ -\beta \end{pmatrix} = \begin{pmatrix} \gamma^{-1} \\ 0 \end{pmatrix}$$



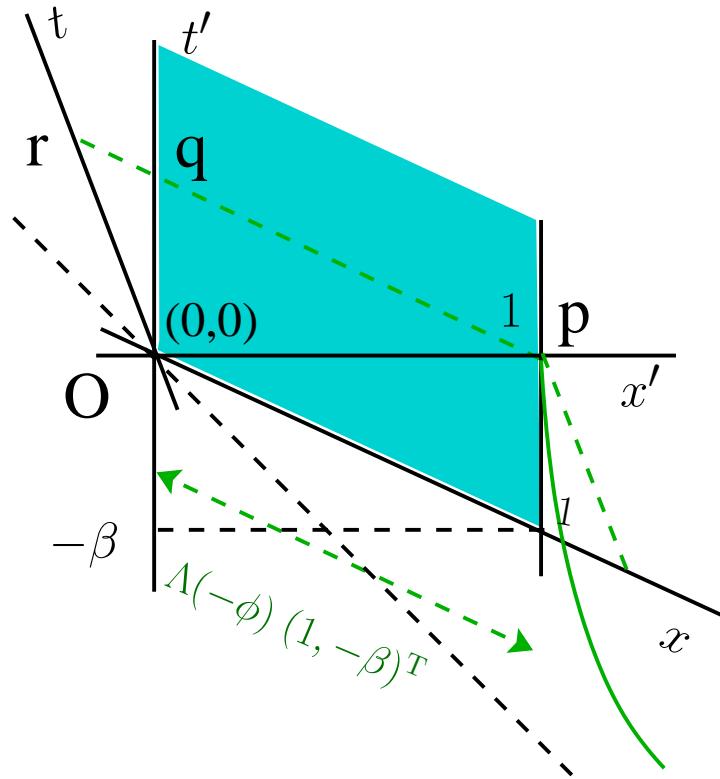
SR: worldsheet space contraction



$$\sqrt{\Delta s^2(q, p)} = \gamma^{-1} < 1$$



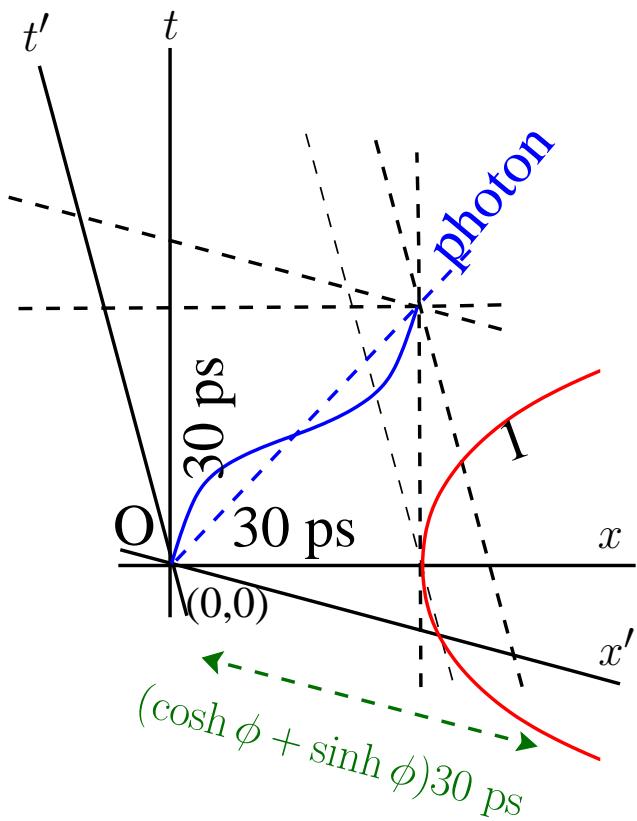
SR: worldsheet space contraction



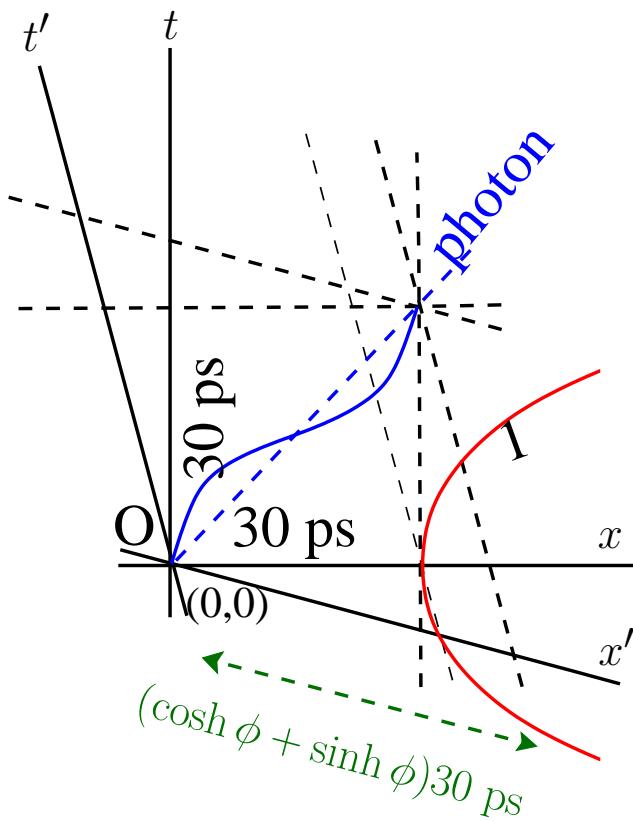
$$\sqrt{\Delta s^2(q, p)} = \gamma^{-1} < 1 \quad \text{worldsheet "space contraction"}$$



SR: Doppler shift

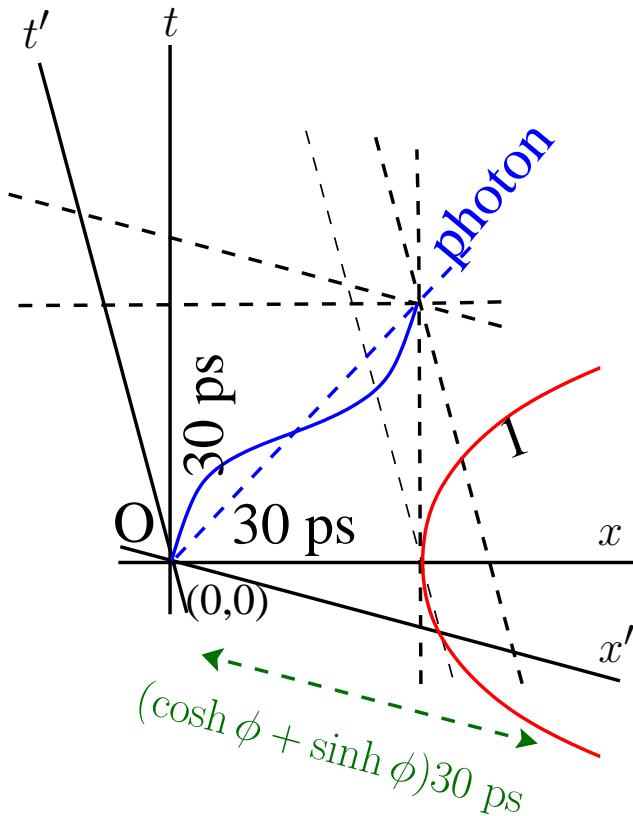


SR: Doppler shift



see photon spacetime path calculation

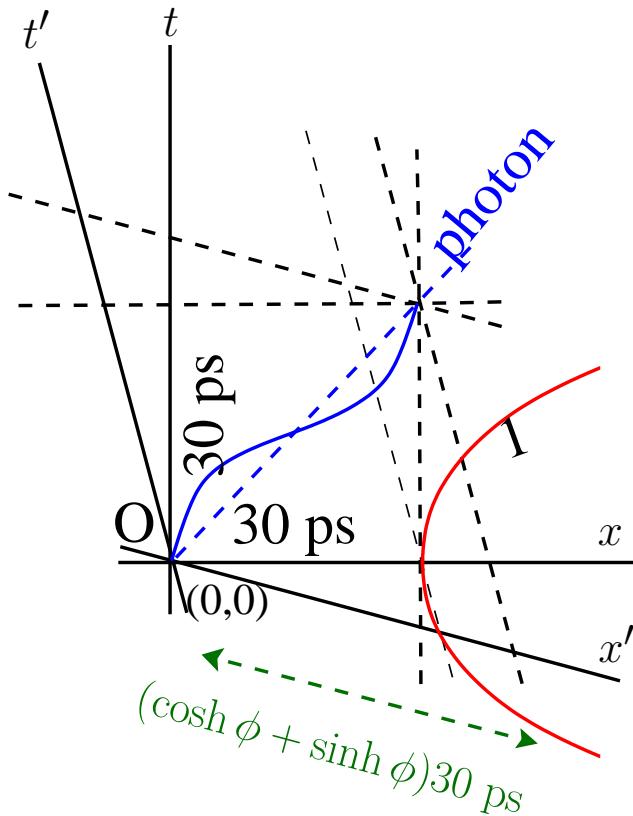
SR: Doppler shift



see photon spacetime path calculation

$$x' = (\cosh \phi + \sinh \phi)t$$

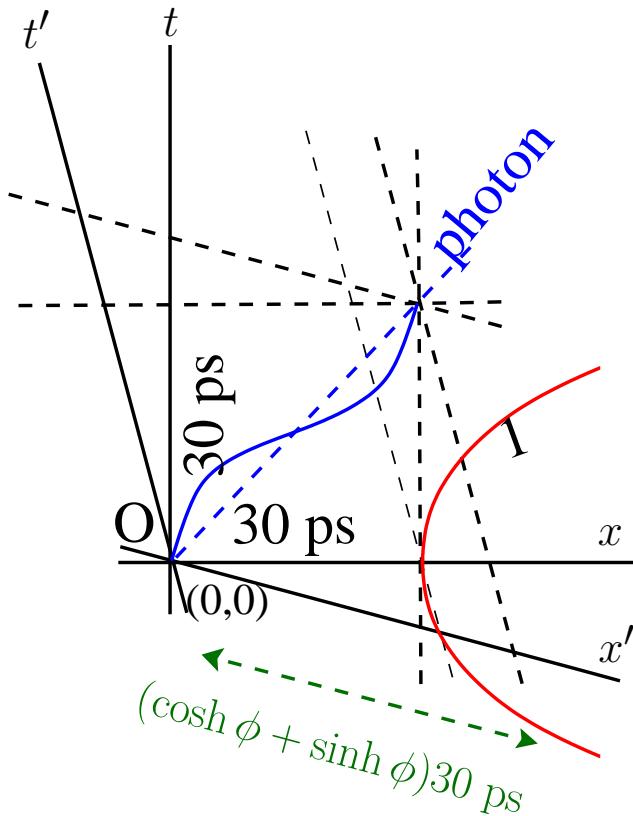
SR: Doppler shift



see photon spacetime path calculation

$$x' = (\cosh \phi + \sinh \phi)x$$

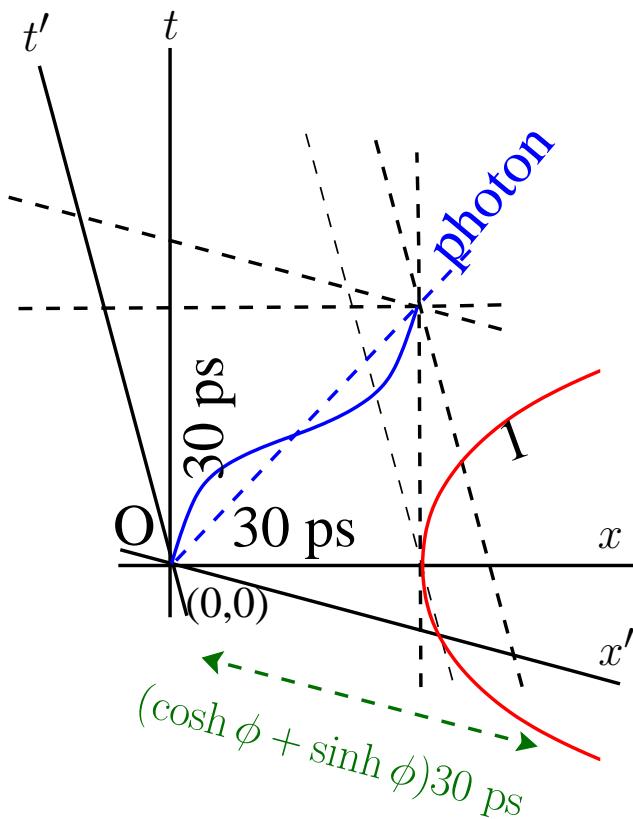
SR: Doppler shift



see photon spacetime path calculation

$$x'/x = \cosh \phi + \sinh \phi$$

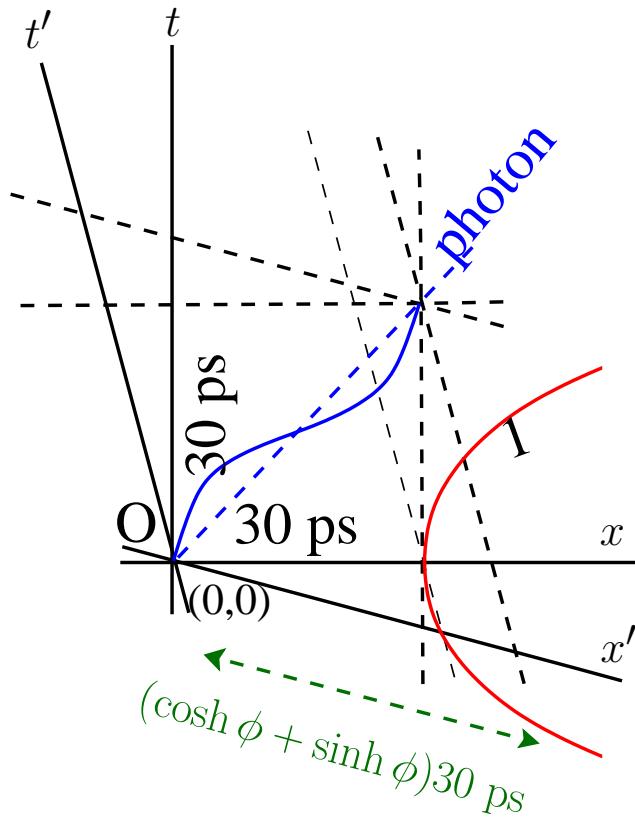
SR: Doppler shift



see photon spacetime path calculation

$$x'/x = \gamma + \beta\gamma$$

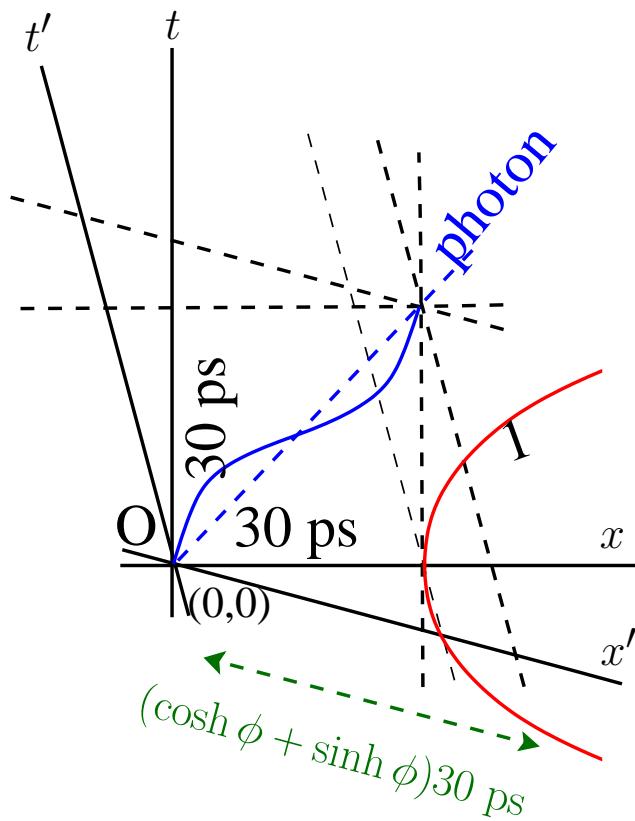
SR: Doppler shift



see photon spacetime path calculation

$$x'/x = \gamma(1 + \beta)$$

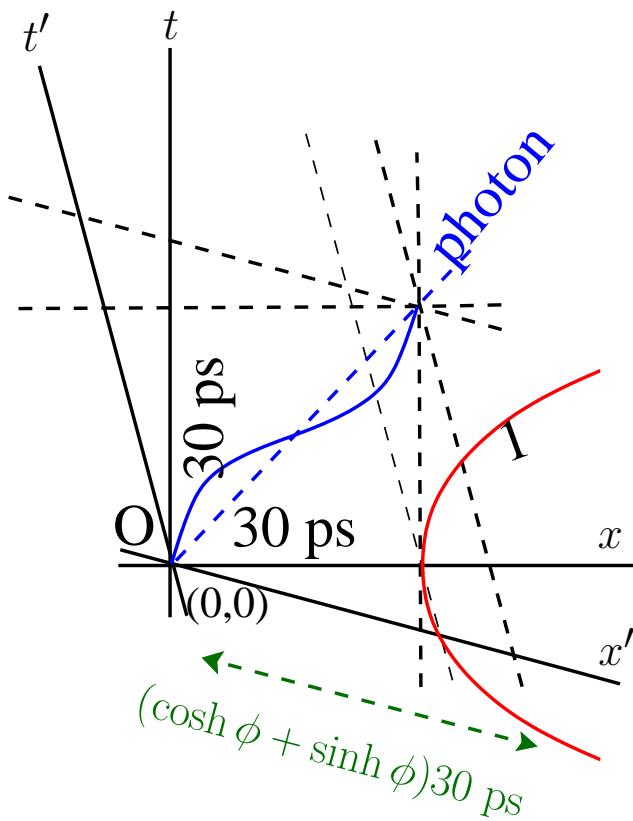
SR: Doppler shift



see photon spacetime path calculation

$$x'/x = \gamma(1 + \beta) = \frac{1+\beta}{\sqrt{1-\beta^2}}$$

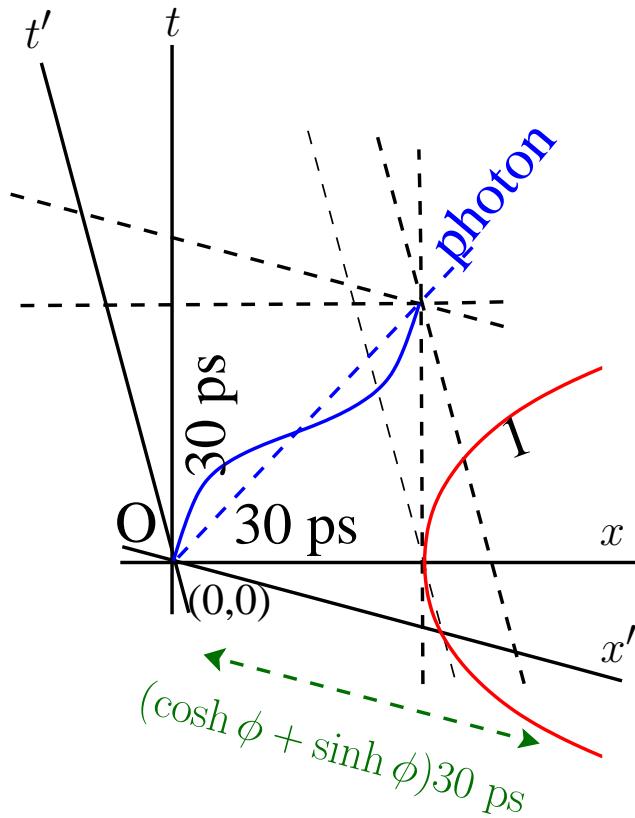
SR: Doppler shift



see photon spacetime path calculation

$$x'/x = \gamma(1 + \beta) = \frac{\sqrt{(1+\beta)^2}}{\sqrt{(1-\beta)(1+\beta)}}$$

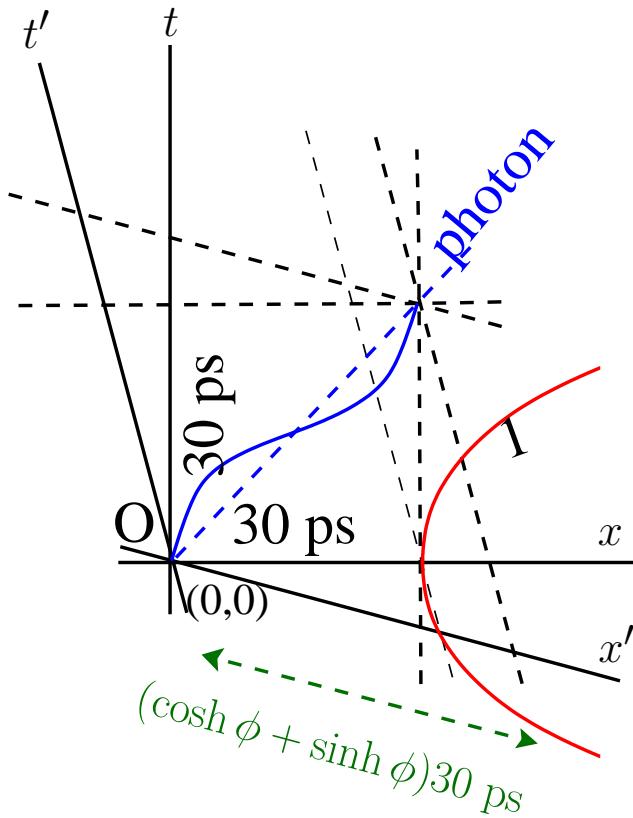
SR: Doppler shift



see photon spacetime path calculation

$$x'/x = \gamma(1 + \beta) = \sqrt{\frac{1+\beta}{1-\beta}}$$

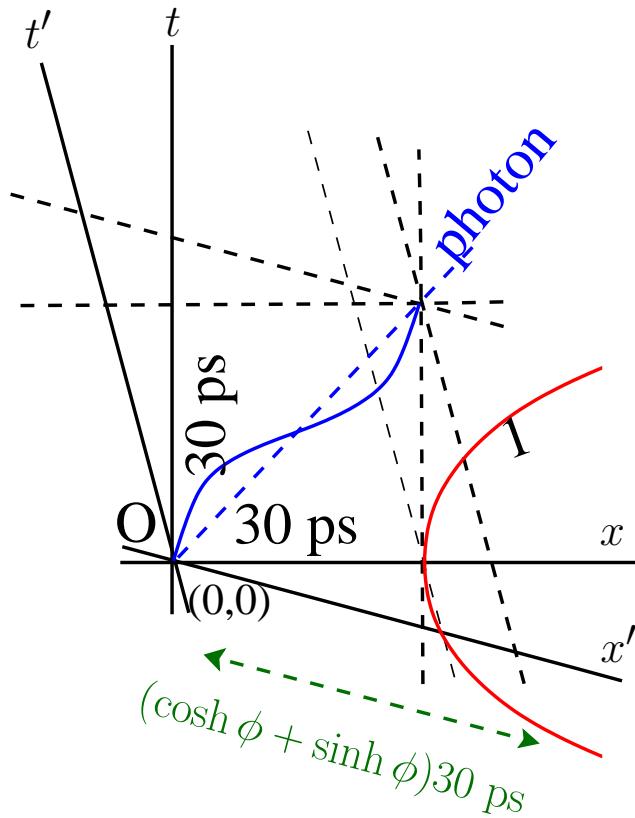
SR: Doppler shift



see photon spacetime path calculation

$$1 + z := \lambda'/\lambda = \gamma(1 + \beta) \equiv \sqrt{\frac{1+\beta}{1-\beta}} \quad \text{redshift}$$

SR: Doppler shift



see photon spacetime path calculation

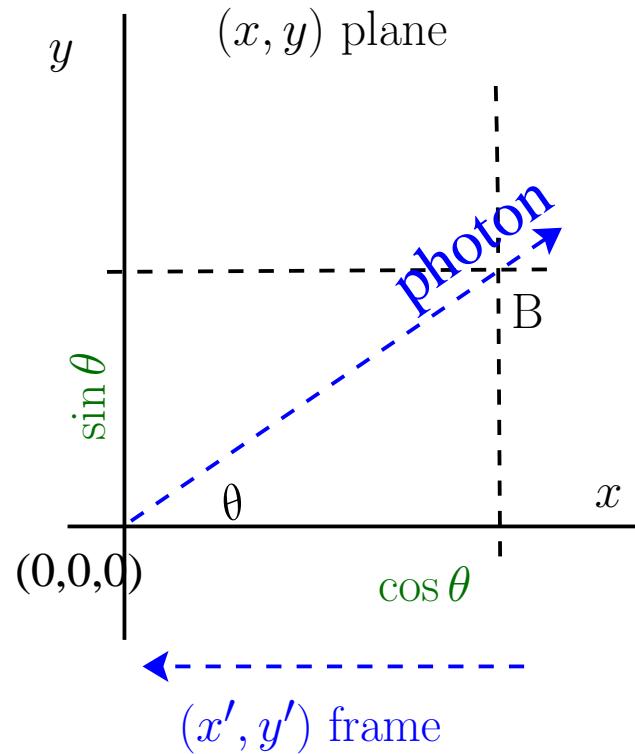
$$1 + z := \lambda'/\lambda = \gamma(1 + \beta) \equiv \sqrt{\frac{1+\beta}{1-\beta}} \quad \text{redshift}$$

\Rightarrow when $\beta \ll 1$, $z \approx \beta$



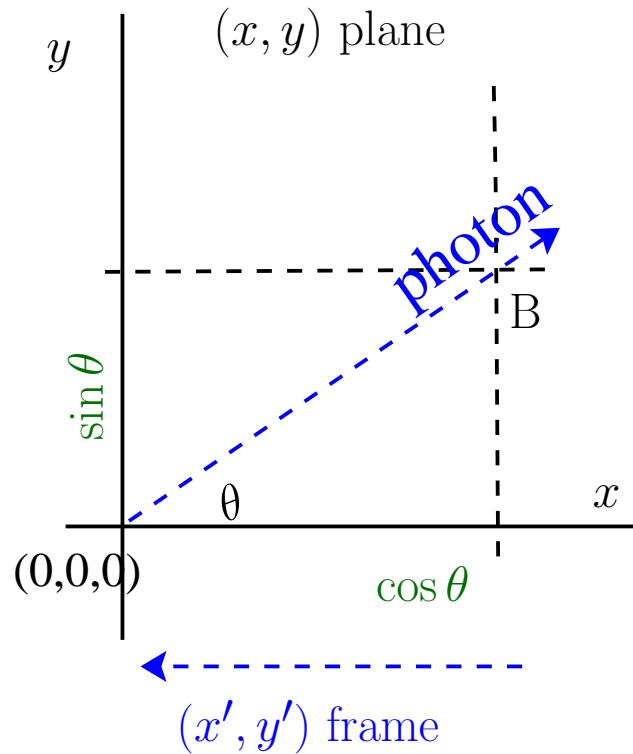


SR: relativistic aberration





SR: relativistic aberration



event B: $(x, y, t) = (\cos \theta, \sin \theta, 1)$





SR: relativistic aberration

event B: $(x, y, t) = (\cos \theta, \sin \theta, 1)$





SR: relativistic aberration

event B: $(x, y, t) = (\cos \theta, \sin \theta, 1)$

$$\Lambda^{-1} \begin{pmatrix} \cos \theta \\ \sin \theta \\ 1 \end{pmatrix}$$





SR: relativistic aberration

event B: $(x, y, t) = (\cos \theta, \sin \theta, 1)$

$$\Lambda^{-1} \begin{pmatrix} \cos \theta \\ \sin \theta \\ 1 \end{pmatrix} = \begin{pmatrix} \gamma & 0 & \beta\gamma \\ 0 & 1 & 0 \\ \beta\gamma & 0 & \gamma \end{pmatrix} \begin{pmatrix} \cos \theta \\ \sin \theta \\ 1 \end{pmatrix}$$





SR: relativistic aberration

event B: $(x, y, t) = (\cos \theta, \sin \theta, 1)$

$$\Lambda^{-1} \begin{pmatrix} \cos \theta \\ \sin \theta \\ 1 \end{pmatrix} = \begin{pmatrix} \gamma \cos \theta + \beta \gamma \\ \sin \theta \\ \beta \gamma \cos \theta + \gamma \end{pmatrix}$$





SR: relativistic aberration

event B: $(x, y, t) = (\cos \theta, \sin \theta, 1)$

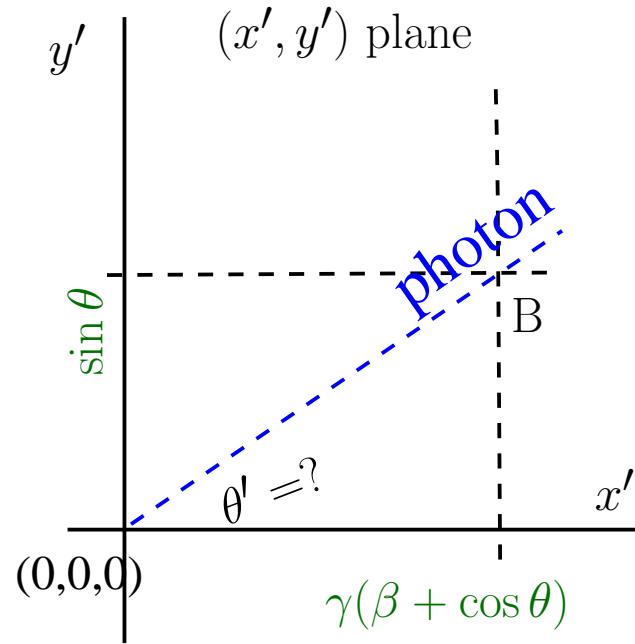
$$\Lambda^{-1} \begin{pmatrix} \cos \theta \\ \sin \theta \\ 1 \end{pmatrix} = \begin{pmatrix} \gamma(\cos \theta + \beta) \\ \sin \theta \\ \gamma(1 + \beta \cos \theta) \end{pmatrix}$$





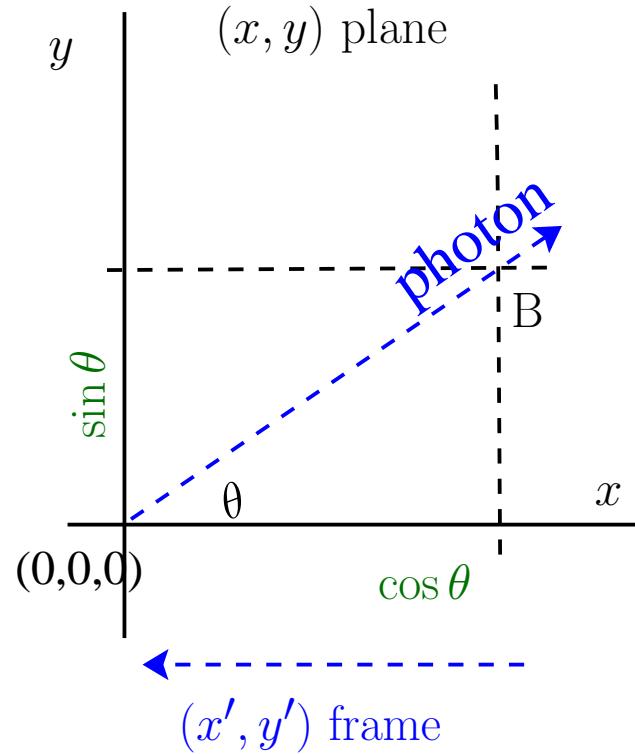
SR: relativistic aberration

event B: $(x, y, t) = (\cos \theta, \sin \theta, 1)$



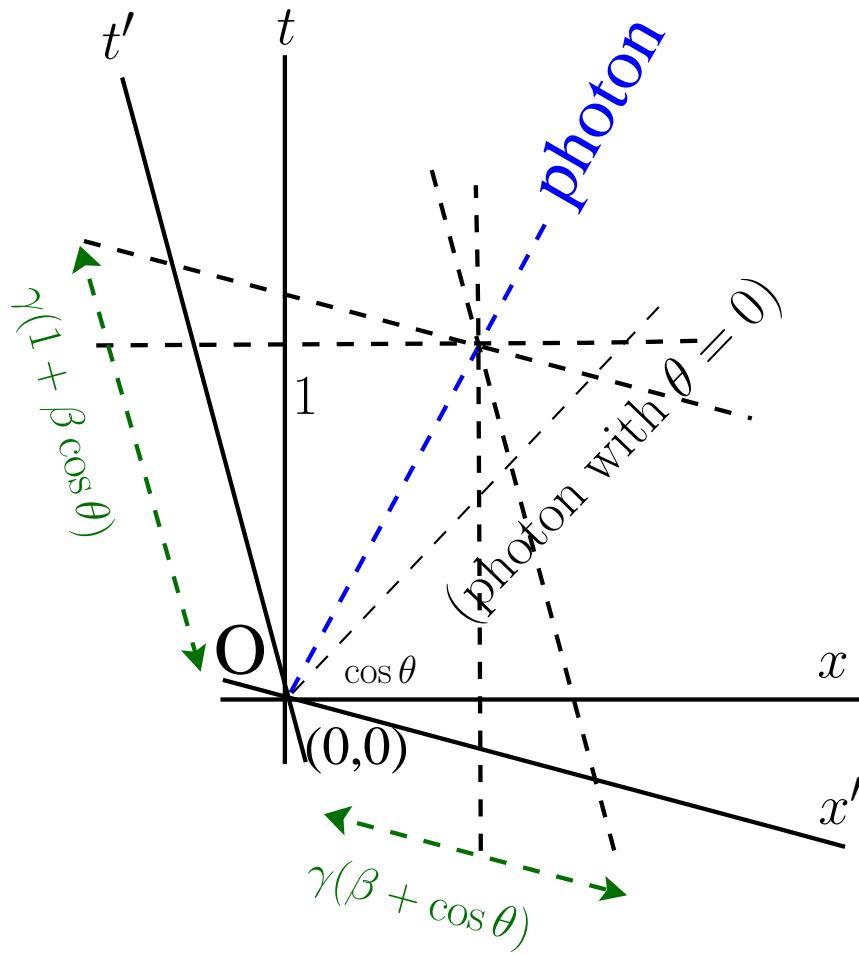
SR: relativistic aberration

event B: $(x, y, t) = (\cos \theta, \sin \theta, 1)$



SR: relativistic aberration

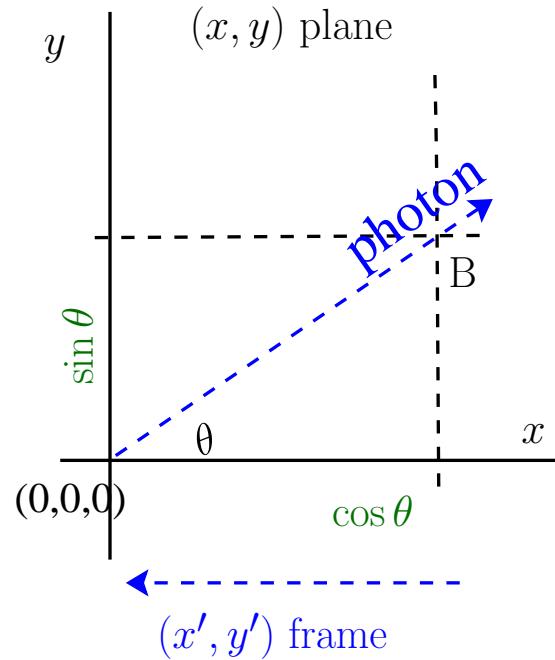
event B: $(x, y, t) = (\cos \theta, \sin \theta, 1)$





SR: relativistic aberration

event B: $(x, y, t) = (\cos \theta, \sin \theta, 1)$



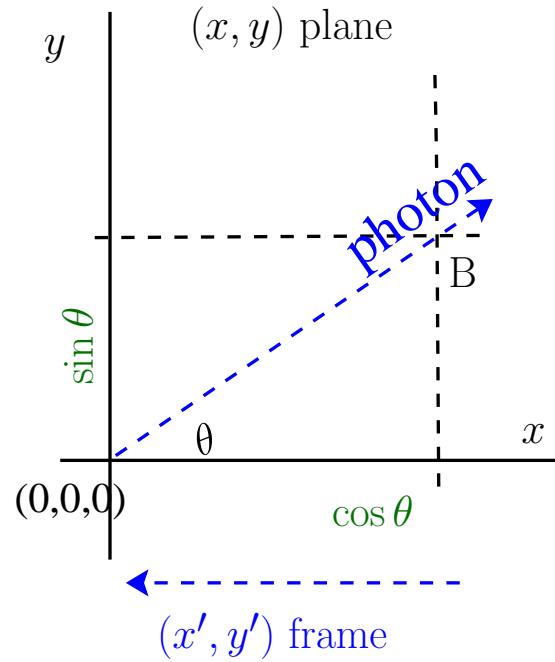
$$\tan \theta' = \frac{\sin \theta}{\gamma(\beta + \cos \theta)}$$





SR: relativistic aberration

event B: $(x, y, t) = (\cos \theta, \sin \theta, 1)$



$$\tan \theta' = \frac{\sin \theta}{\gamma(\beta + \cos \theta)} < \tan \theta \text{ if } 0 < \beta < 1$$

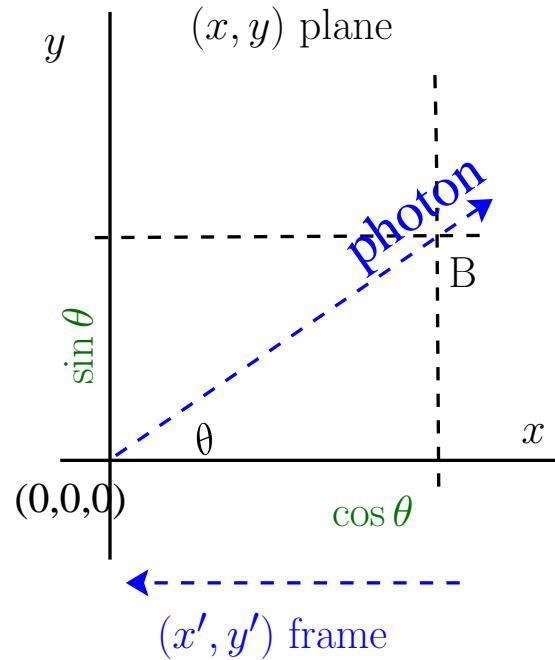
w:Relativistic aberration





SR: relativistic aberration

event B: $(x, y, t) = (\cos \theta, \sin \theta, 1)$



$$\tan \theta' = \frac{\sin \theta}{\gamma(\beta + \cos \theta)} < \tan \theta \text{ if } 0 < \beta < 1$$

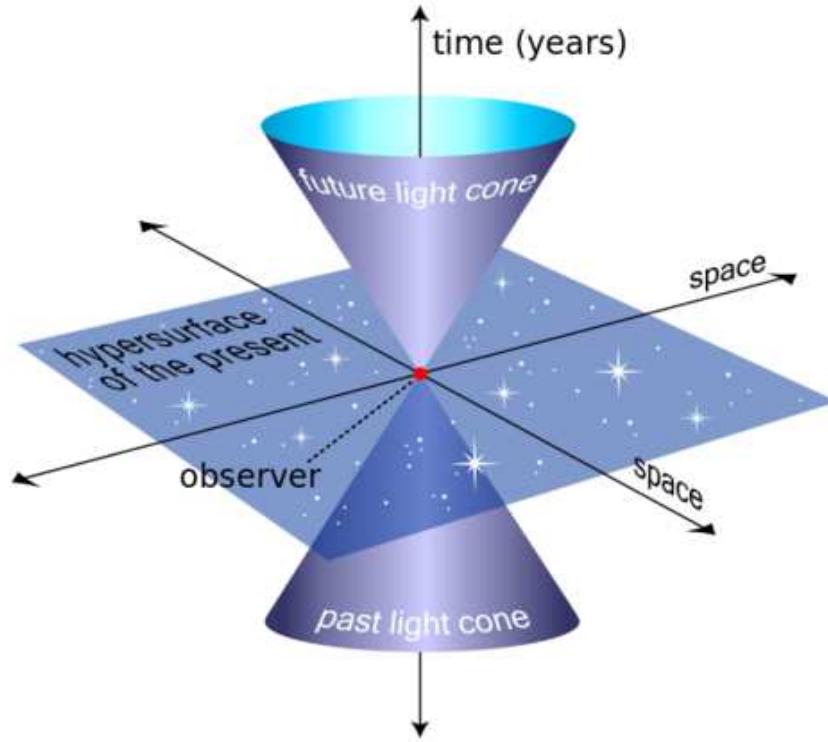
w:Relativistic aberration

⇒ relativistic beaming, e.g. AGN jets



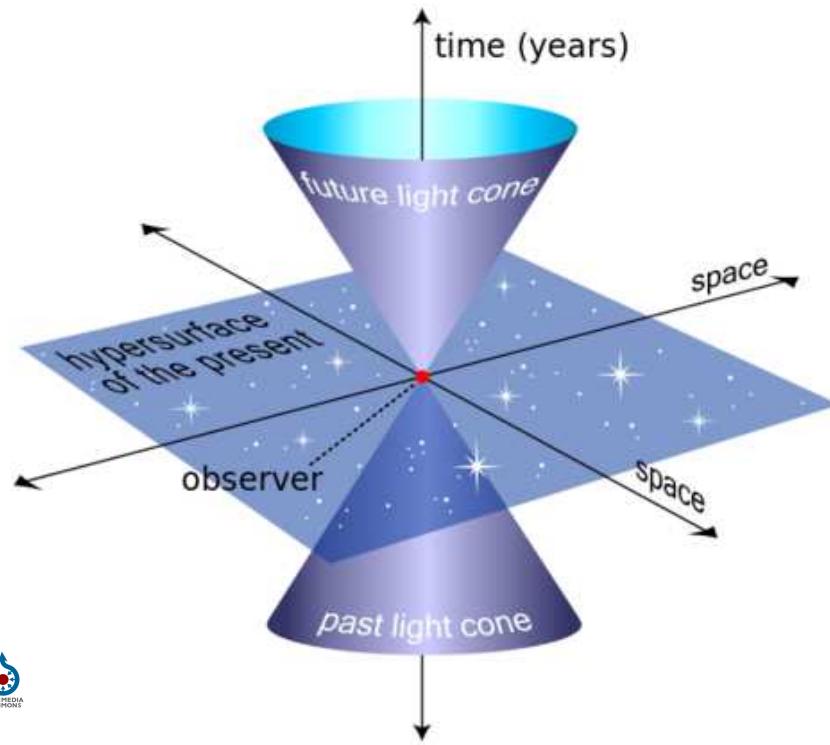


SR: world line





SR: world line



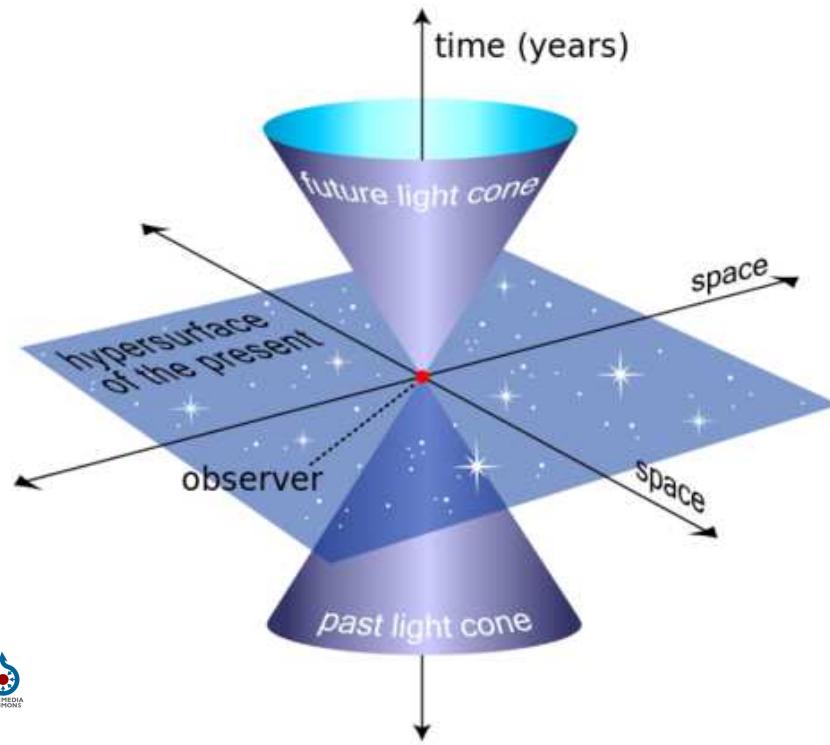
lightlike interval = null interval: $(\Delta s)^2 = 0$

spacetime =





SR: world line



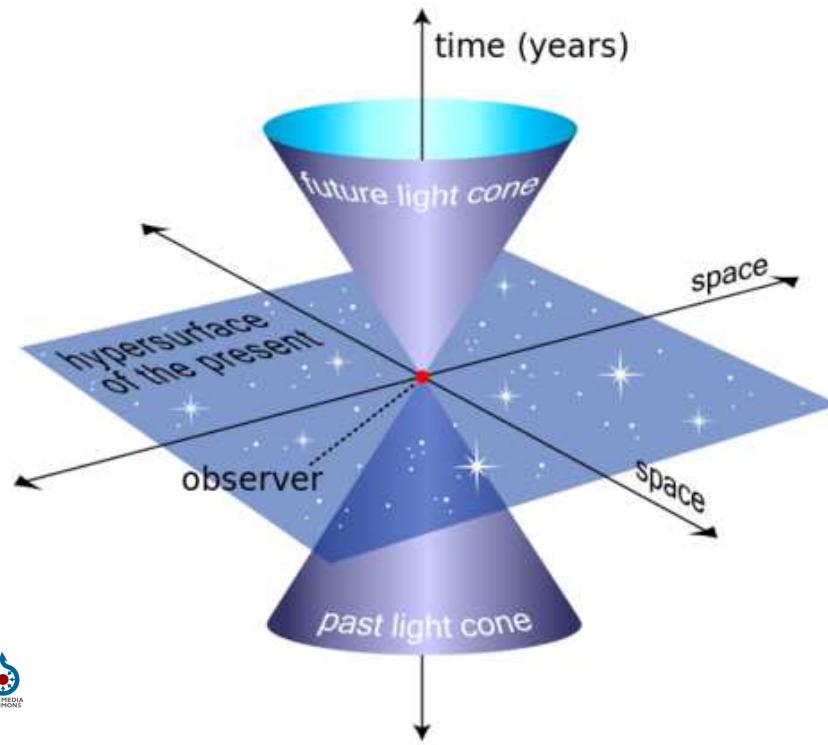
lightlike interval = null interval: $(\Delta s)^2 = 0$

spacetime = on past w:light cone + inside past light cone





SR: world line



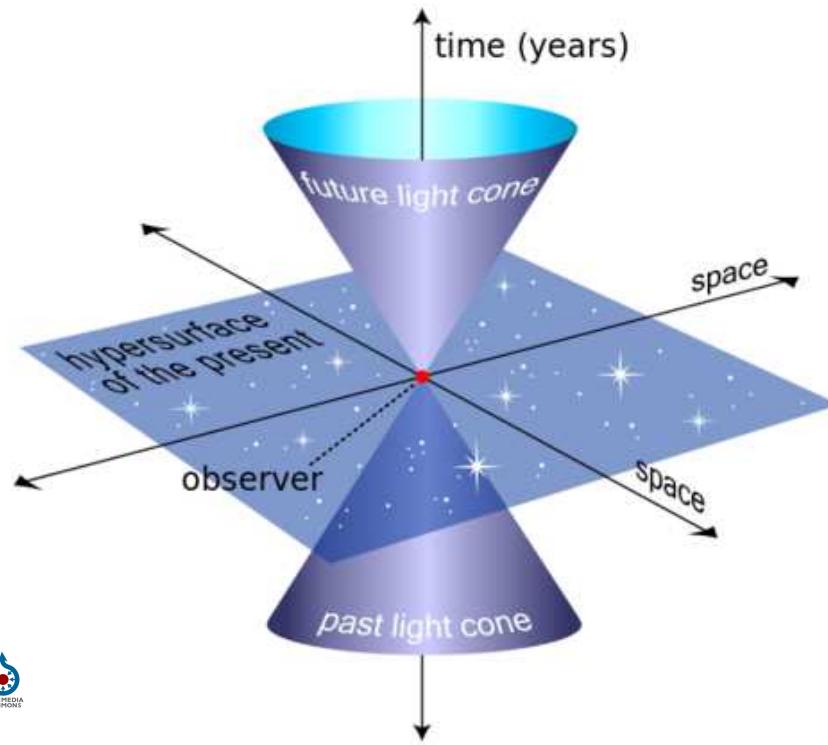
lightlike interval = null interval: $(\Delta s)^2 = 0$

spacetime = on past w:light cone + inside past light cone
+ on future light cone + inside future light cone





SR: world line



lightlike interval = null interval: $(\Delta s)^2 = 0$

spacetime = on past w:**light cone** + inside past light cone

+ on future light cone + inside future light cone

+ elsewhere





SR: world line

Lorentz transform of world line





SR: world line

Lorentz transform of world line





SR: world line

Lorentz transform of world line



- coordinate time in spacetime model \neq time in your brain (thinking)





SR: world line

Lorentz transform of world line



- coordinate time in spacetime model \neq time in your brain (thinking)
- $\frac{dt}{dt_{\text{thinking}}}$ can be positive or negative





SR: world line

Lorentz transform of world line



- coordinate time in spacetime model \neq time in your brain (thinking)
- $\frac{dt}{d\lambda}$ can be positive or negative, λ arbitrary real parameter





SR: world line

Lorentz transform of world line



- coordinate time in spacetime model \neq time in your brain (thinking)
- $\frac{dt}{d\lambda}$ can be positive or negative, λ arbitrary real parameter
- “elsewhere” spacetime events can change from past to future even though $\frac{dt}{d\lambda} > 0$





SR: world line

Lorentz transform of world line



- coordinate time in spacetime model \neq time in your brain (thinking)
- $\frac{dt}{d\lambda}$ can be positive or negative, λ arbitrary real parameter
- “elsewhere” spacetime events can change from past to future even though $\frac{dt}{d\lambda} > 0$
- w:proper time $\tau :=$ time along a worldline measured by clock following that worldline





SR: world line

Lorentz transform of world line

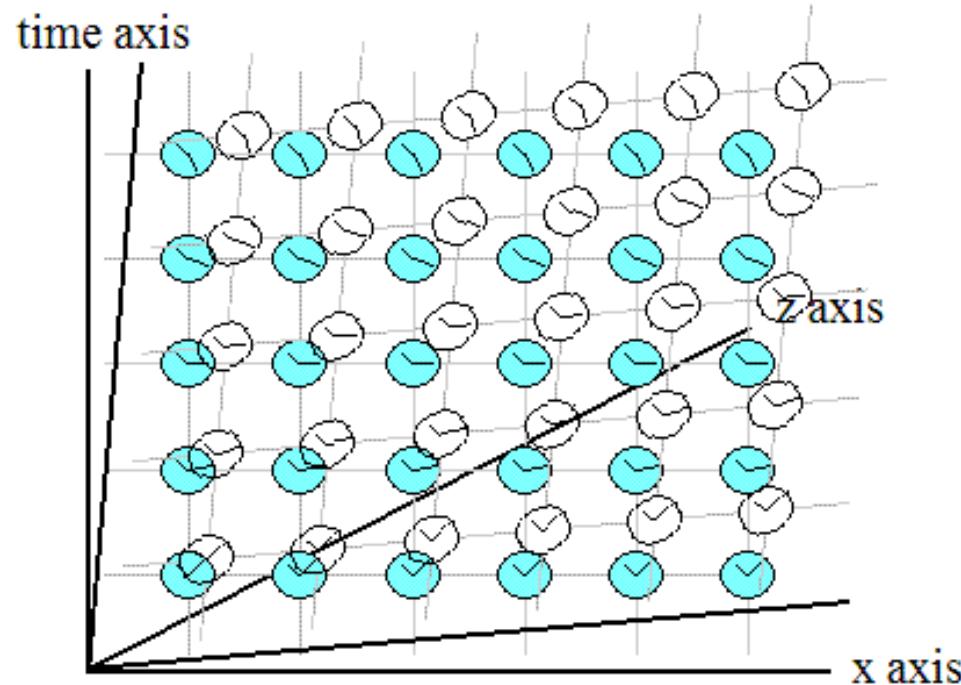


- coordinate time in spacetime model \neq time in your brain (thinking)
- $\frac{dt}{d\lambda}$ can be positive or negative, λ arbitrary real parameter
- “elsewhere” spacetime events can change from past to future even though $\frac{dt}{d\lambda} > 0$
- w:proper time $\tau :=$ time along a worldline measured by clock following that worldline
- often $d\tau$ is useful for integrating





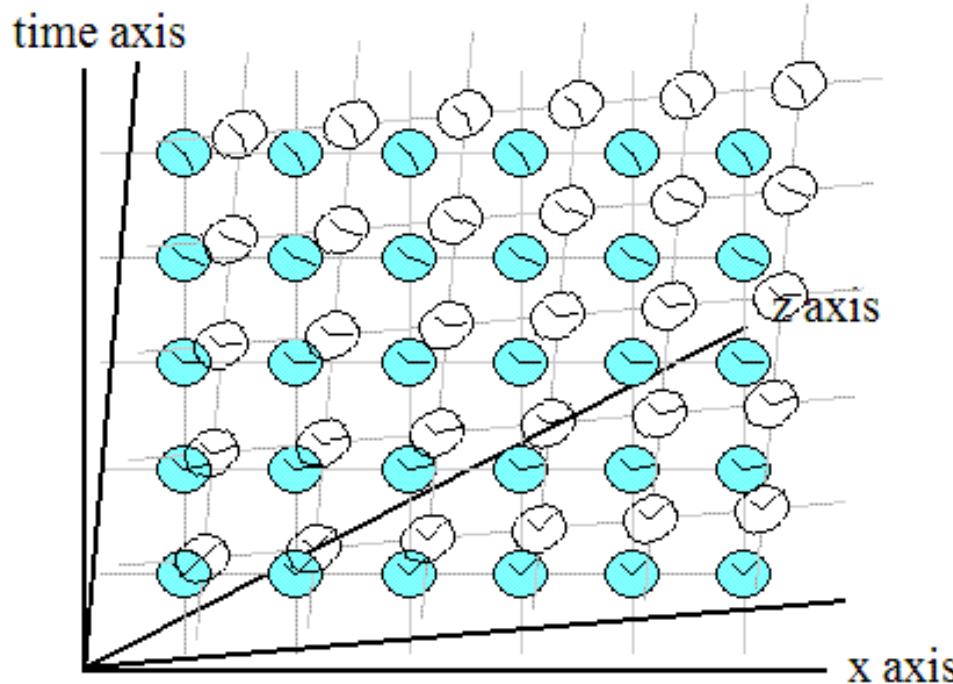
SR: Rietdijk–Putnam–Penrose p.



Relativity shows that the inertial frames of reference of relatively moving objects do not overlap each other.



SR: Rietdijk–Putnam–Penrose p.



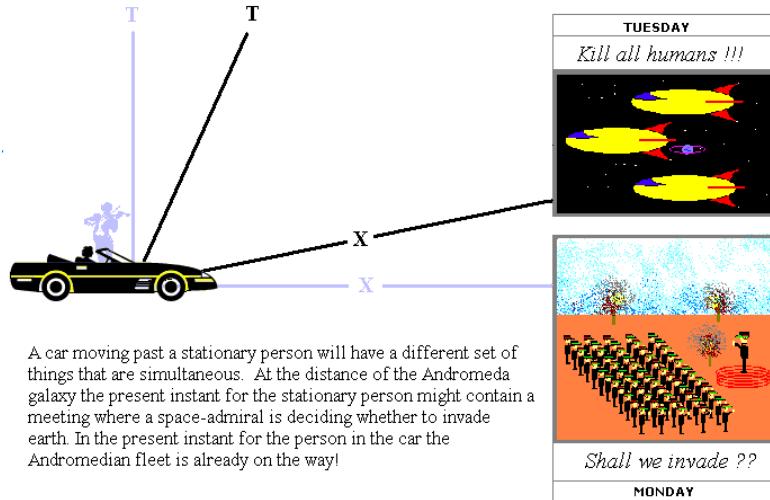
Relativity shows that the inertial frames of reference of relatively moving objects do not overlap each other.

b:Inertialoverlay.GIF

- each observer can synchronise clocks + rods

SR: Rietdijk–Putnam–Penrose p.

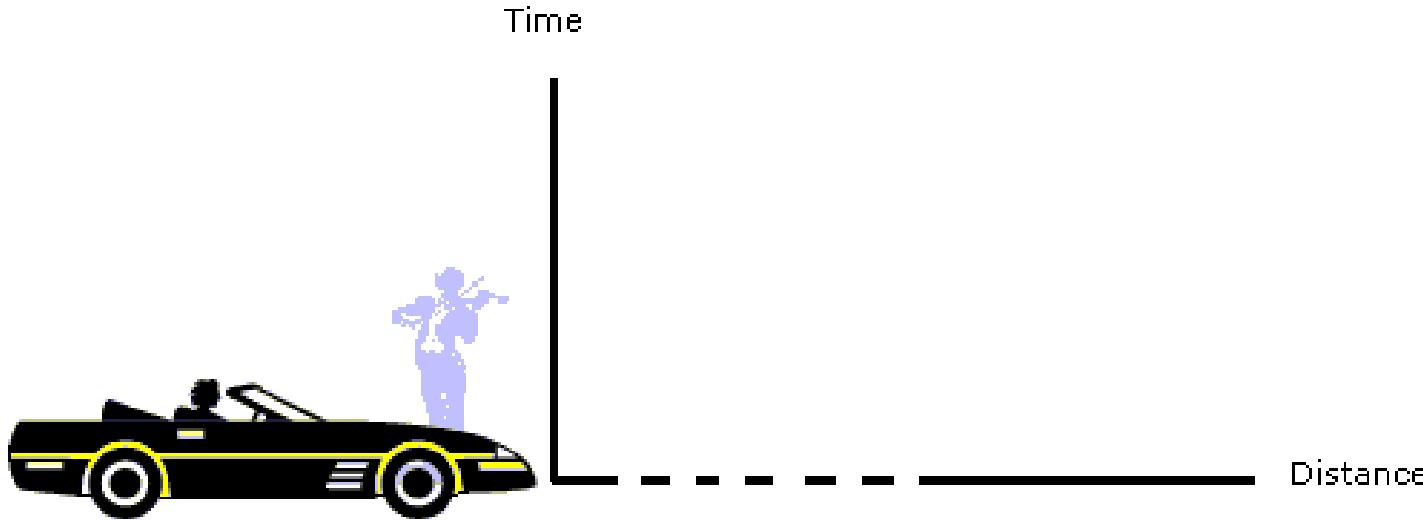
The Andromeda Paradox



w:Rietdijk-Putnam argument b:Rel2.gif



SR: Rietdijk–Putnam–Penrose p.



For the car driver the stationary man and the invasion fleet are all events in the present moment.

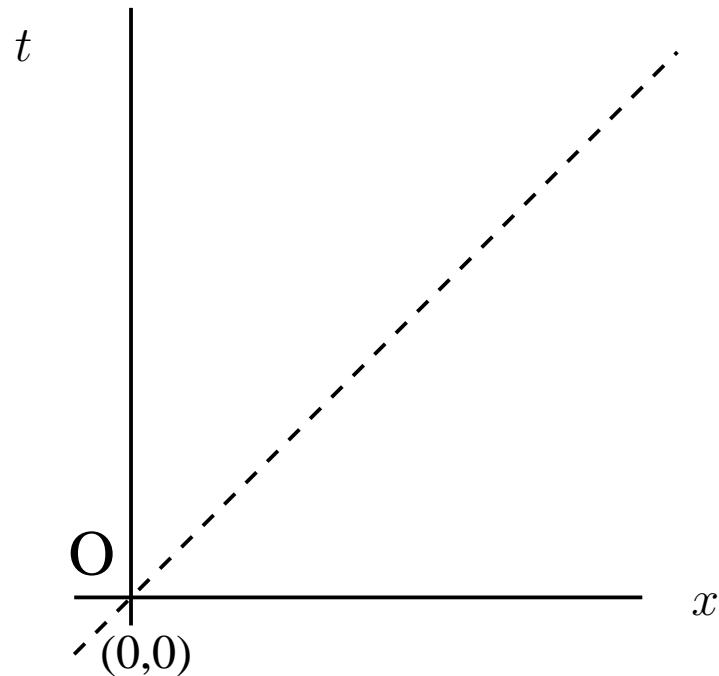


w:Rietdijk–Putnam argument b:Rel3.gif





SR: tachyons and causality

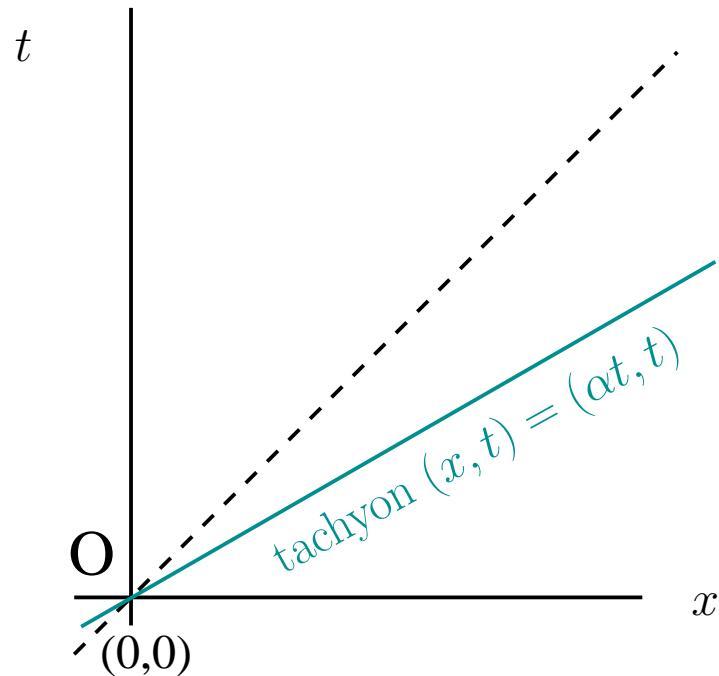


observer “at rest”





SR: tachyons and causality

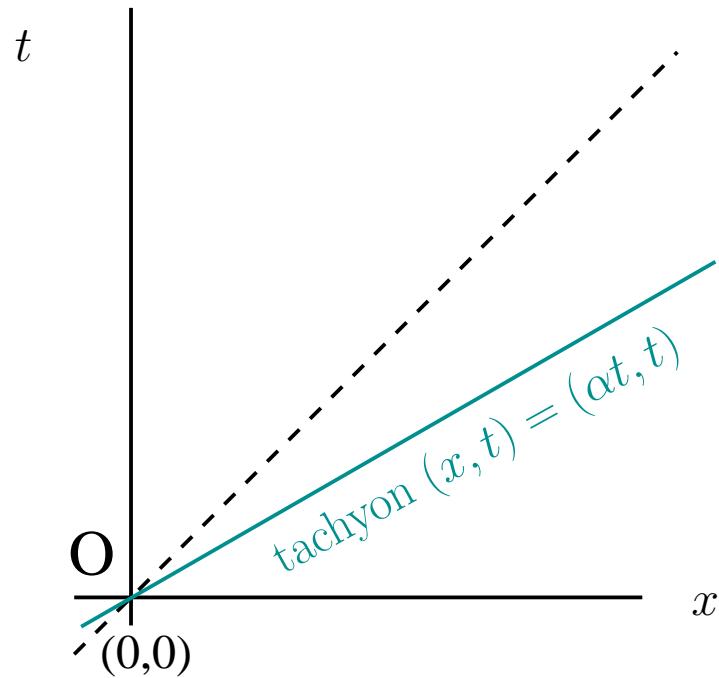


add a tachyon with speed $\alpha > 1$





SR: tachyons and causality



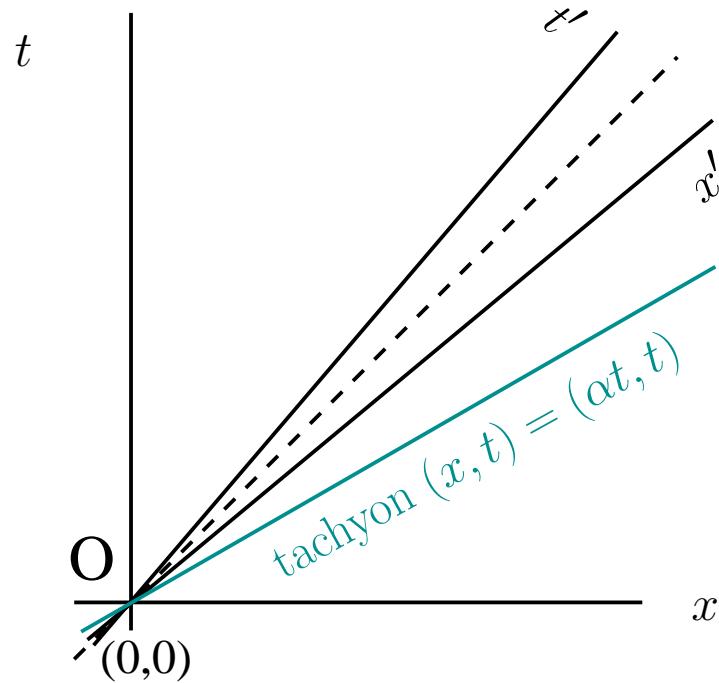
add a tachyon with speed $\alpha > 1$

choose rocket at speed β with $1/\alpha < \beta < 1$





SR: tachyons and causality



add a tachyon with speed $\alpha > 1$

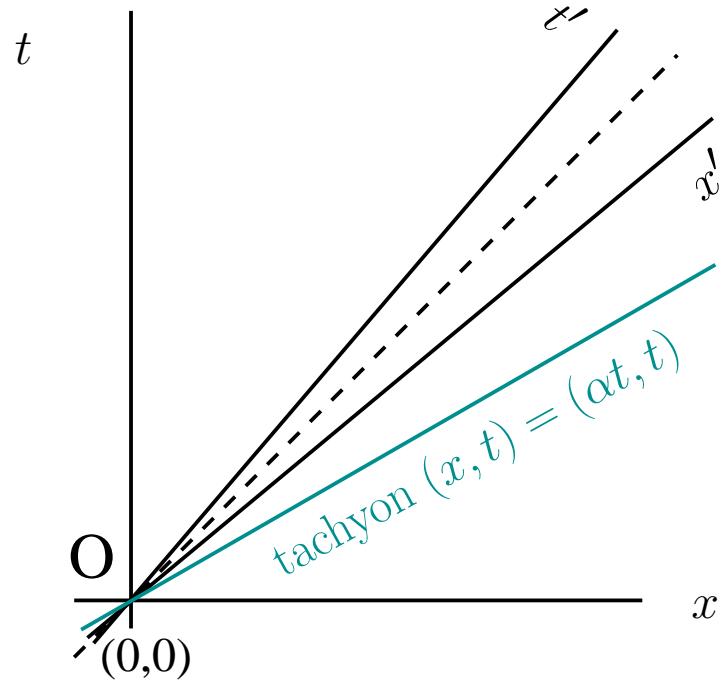
choose rocket at speed β with $1/\alpha < \beta < 1$

add axes x', t' for the rocket





SR: tachyons and causality



add a tachyon with speed $\alpha > 1$

choose rocket at speed β with $1/\alpha < \beta < 1$

add axes x', t' for the rocket

rocket frame: $(\alpha t, t)$ becomes Λ $(\alpha t, t)^T$





SR: tachyons and causality

$$\begin{pmatrix} x' \\ t' \end{pmatrix} = \Lambda \begin{pmatrix} \alpha t \\ t \end{pmatrix} =$$





SR: tachyons and causality

$$\begin{pmatrix} x' \\ t' \end{pmatrix} = \Lambda \begin{pmatrix} \alpha t \\ t \end{pmatrix} = \begin{pmatrix} \gamma\alpha t - \beta\gamma t \\ -\alpha\beta\gamma t + \gamma t \end{pmatrix}$$





SR: tachyons and causality

$$\begin{pmatrix} x' \\ t' \end{pmatrix} = \Lambda \begin{pmatrix} \alpha t \\ t \end{pmatrix} = \gamma t \begin{pmatrix} \alpha - \beta \\ -\alpha\beta + 1 \end{pmatrix}$$





SR: tachyons and causality

$$\begin{pmatrix} x' \\ t' \end{pmatrix} = \Lambda \begin{pmatrix} \alpha t \\ t \end{pmatrix} = \gamma t \begin{pmatrix} \alpha - \beta \\ -\alpha\beta + 1 \end{pmatrix}$$

$$x' = \gamma t(\alpha - \beta) > 0 \text{ since } \alpha > 1 > \beta$$





SR: tachyons and causality

$$\begin{pmatrix} x' \\ t' \end{pmatrix} = \Lambda \begin{pmatrix} \alpha t \\ t \end{pmatrix} = \gamma t \begin{pmatrix} \alpha - \beta \\ -\alpha\beta + 1 \end{pmatrix}$$

$x' = \gamma t(\alpha - \beta) > 0$ since $\alpha > 1 > \beta$

$t' = \gamma t(-\alpha\beta + 1) < 0$ since we chose $\beta > 1/\alpha$





SR: tachyons and causality

$$\begin{pmatrix} x' \\ t' \end{pmatrix} = \Lambda \begin{pmatrix} \alpha t \\ t \end{pmatrix} = \gamma t \begin{pmatrix} \alpha - \beta \\ -\alpha\beta + 1 \end{pmatrix}$$

$x' = \gamma t(\alpha - \beta) > 0$ since $\alpha > 1 > \beta$

$t' = \gamma t(-\alpha\beta + 1) < 0$ since we chose $\beta > 1/\alpha$

$dt'/dt < 0$

same sequence of spacetime events = tachyon
spacetime path:

t increases for observer “at rest”,

t' decreases for rocket observer (with $\beta > 1/\alpha$)





SR: tachyons and causality

$$\begin{pmatrix} x' \\ t' \end{pmatrix} = \Lambda \begin{pmatrix} \alpha t \\ t \end{pmatrix} = \gamma t \begin{pmatrix} \alpha - \beta \\ -\alpha\beta + 1 \end{pmatrix}$$

$x' = \gamma t(\alpha - \beta) > 0$ since $\alpha > 1 > \beta$

$t' = \gamma t(-\alpha\beta + 1) < 0$ since we chose $\beta > 1/\alpha$

$dt'/dt < 0$

same sequence of spacetime events = tachyon
spacetime path:

t increases for observer “at rest”,

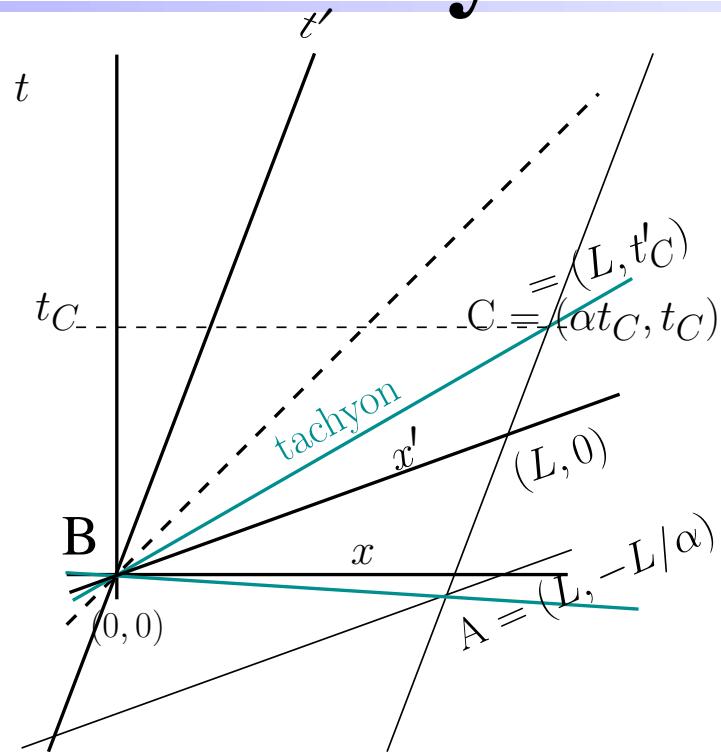
t' decreases for rocket observer (with $\beta > 1/\alpha$)

- observer at rest: tachyon emitted at origin
- rocket: tachyon absorbed at origin



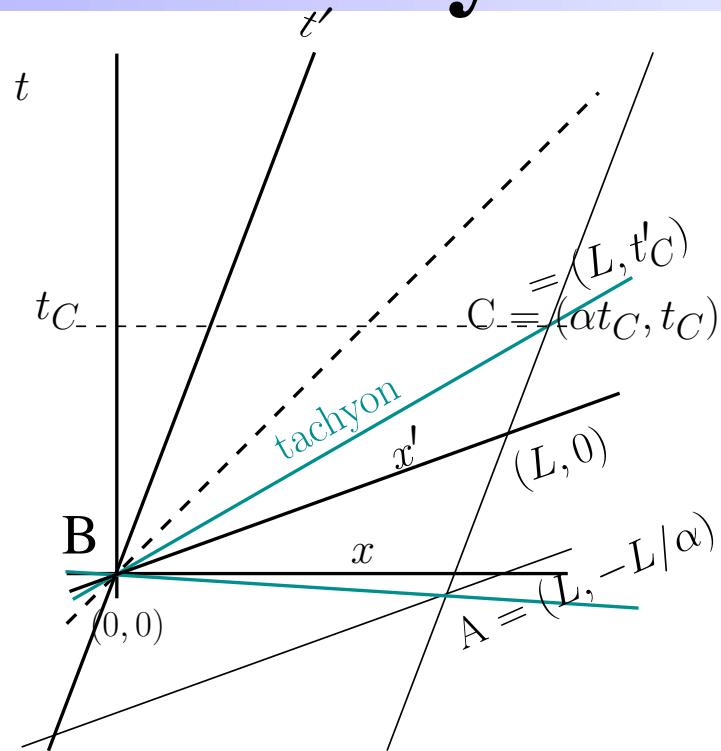


SR: tachyonic antitelephone





SR: tachyonic antitelephone

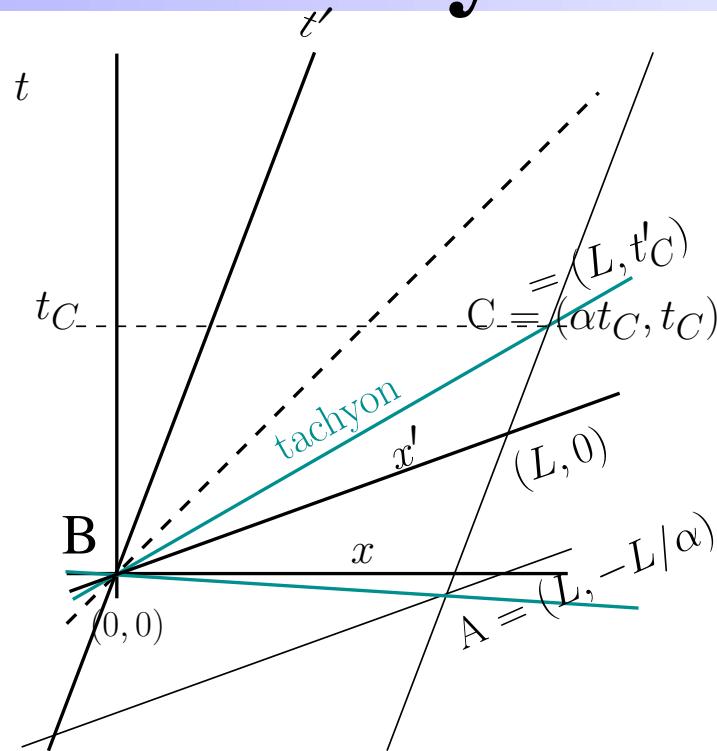


B stationary: (x, t)
frame





SR: tachyonic antitelephone



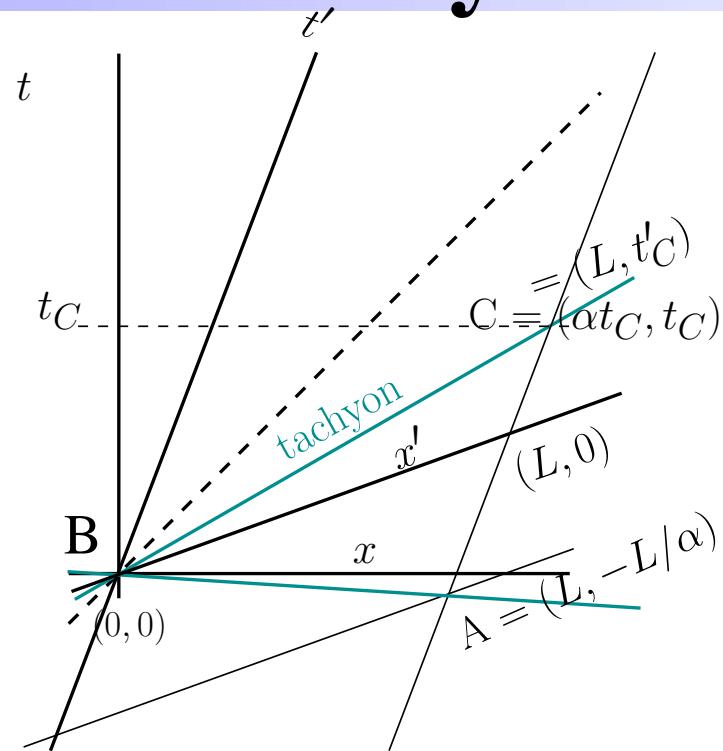
B stationary: (x, t)
frame

A moving at speed β :
 (x', t') frame





SR: tachyonic antitelephone



A: tachyon at $\alpha > 1$ to B

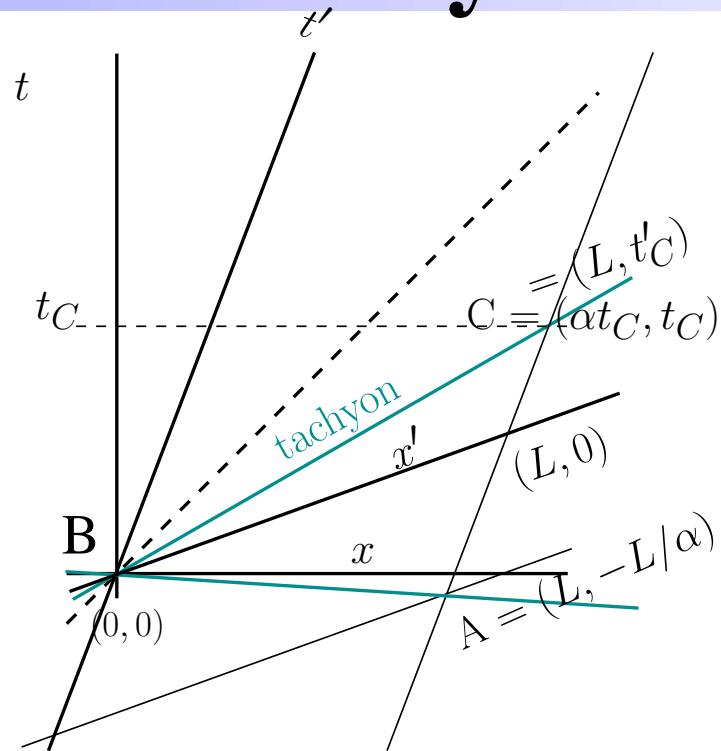
B stationary: (x, t)
frame

A moving at speed β :
 (x', t') frame





SR: tachyonic antitelephone



B: tachyon at $\alpha > 1$ to C

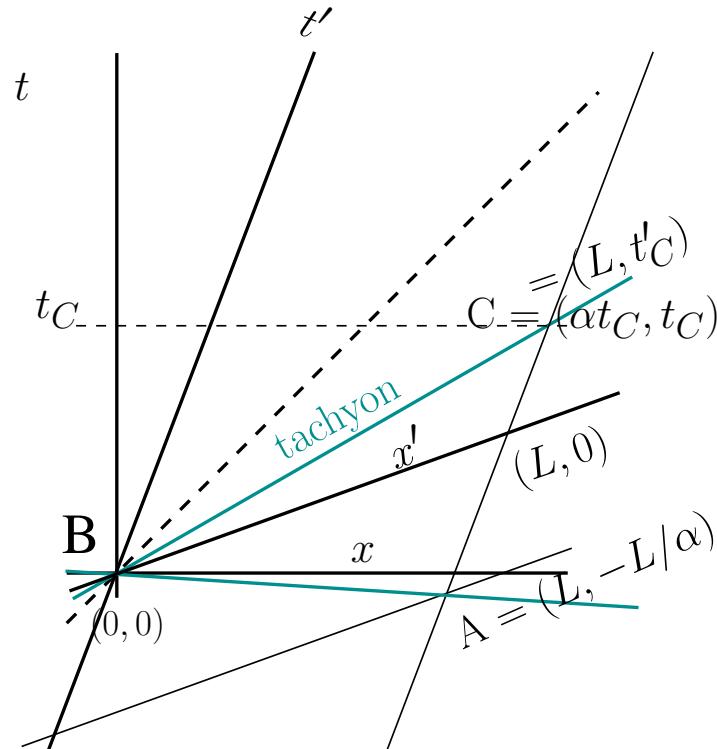
B stationary: (x, t)
frame

A moving at speed β :
 (x', t') frame





SR: tachyonic antitelephone



$$C: \begin{pmatrix} L \\ t'_C \end{pmatrix} = \Lambda \begin{pmatrix} \alpha t_C \\ t_C \end{pmatrix}$$

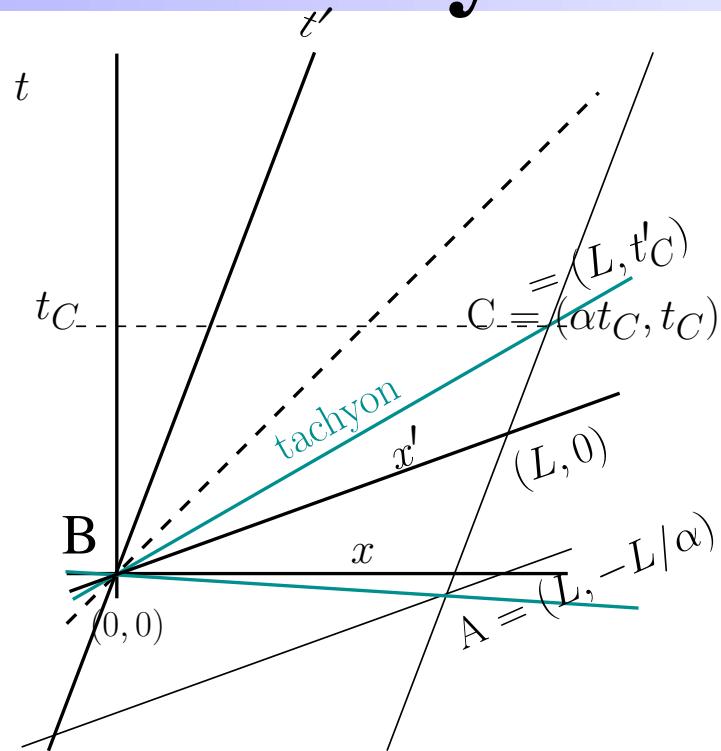
B stationary: (x, t)
frame

A moving at speed β :
 (x', t') frame





SR: tachyonic antitelephone



$$\begin{pmatrix} L \\ t'_C \end{pmatrix} = \gamma t_C \begin{pmatrix} \alpha - \beta \\ -\alpha\beta + 1 \end{pmatrix}$$

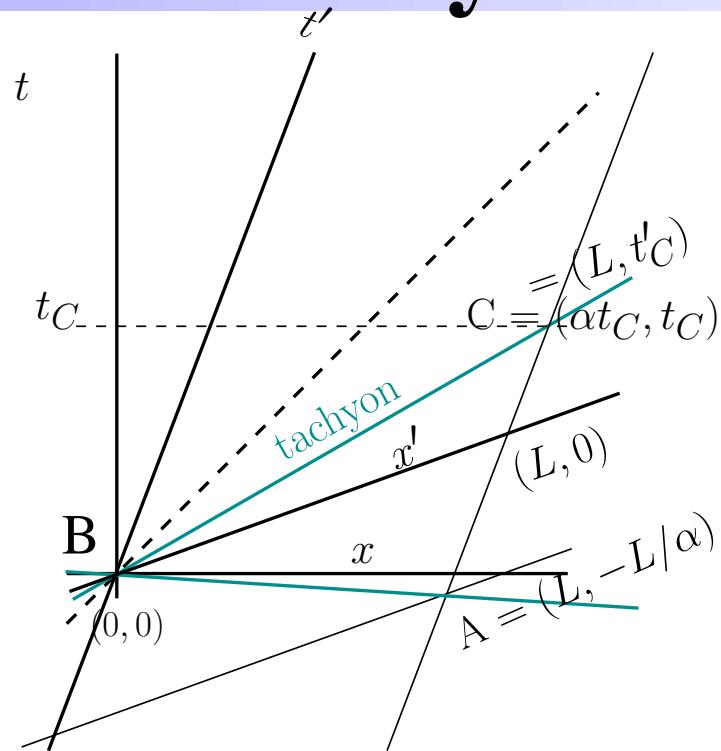
B stationary: (x, t)
frame

A moving at speed β :
 (x', t') frame





SR: tachyonic antitelephone



$$\begin{pmatrix} L \\ t'_C \end{pmatrix} = \gamma t_C \begin{pmatrix} \alpha - \beta \\ -\alpha\beta + 1 \end{pmatrix}$$

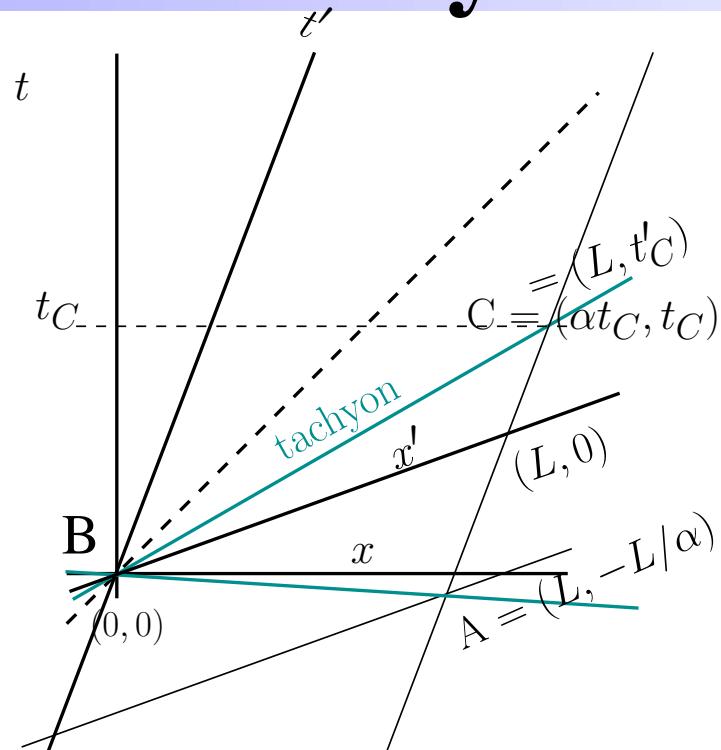
$$t'_C = \gamma t_C (1 - \alpha\beta)$$

B stationary: (x, t)
 frame
 A moving at speed β :
 (x', t') frame





SR: tachyonic antitelephone



$$\begin{pmatrix} L \\ t'_C \end{pmatrix} = \gamma t_C \begin{pmatrix} \alpha - \beta \\ -\alpha\beta + 1 \end{pmatrix}$$

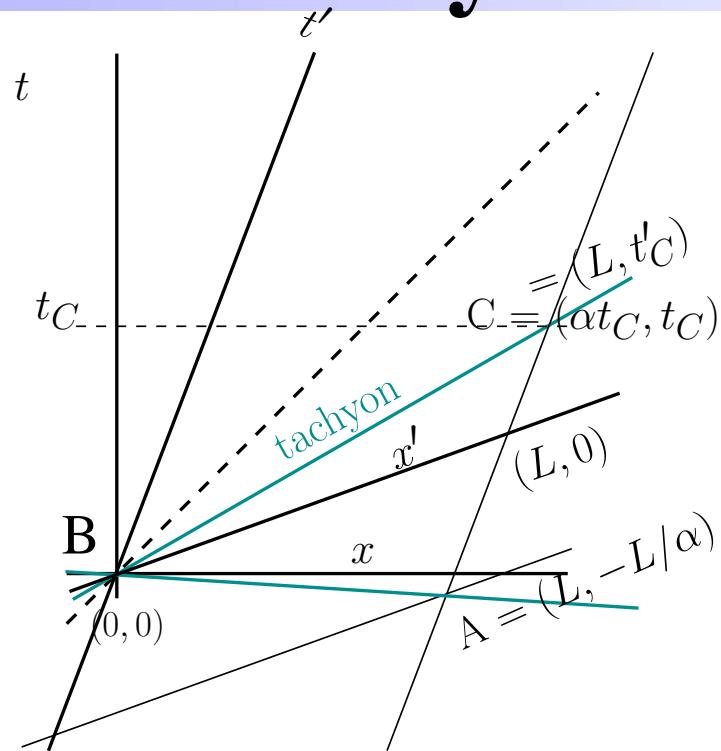
$$t'_C = \gamma \frac{L}{\gamma(\alpha-\beta)}(1 - \alpha\beta)$$

B stationary: (x, t)
frame
A moving at speed β :
 (x', t') frame





SR: tachyonic antitelephone



$$\begin{pmatrix} L \\ t'_C \end{pmatrix} = \gamma t_C \begin{pmatrix} \alpha - \beta \\ -\alpha\beta + 1 \end{pmatrix}$$

$$t'_C = L \frac{1 - \alpha\beta}{\alpha - \beta}$$

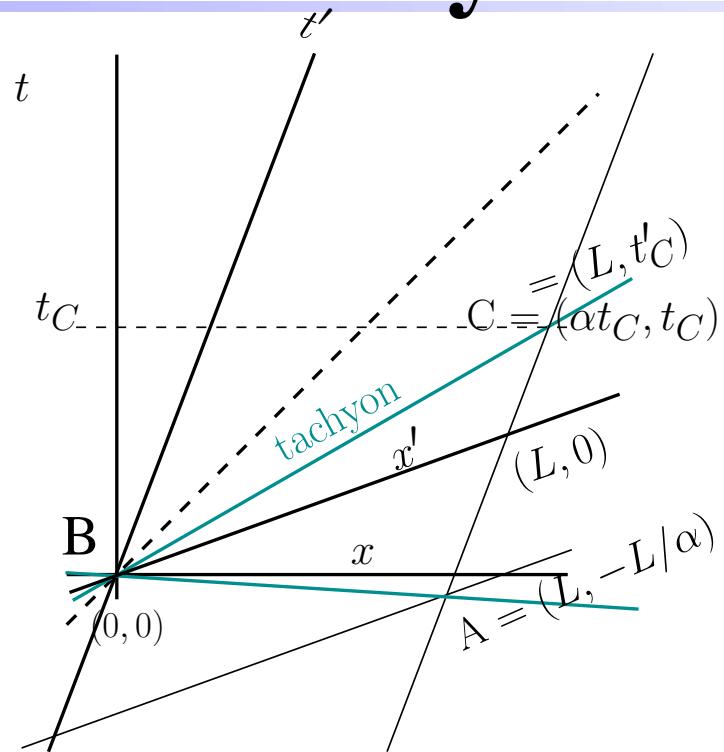
B stationary: (x, t)
frame

A moving at speed β :
 (x', t') frame





SR: tachyonic antitelephone



B stationary: (x, t)
frame
A moving at speed β :
 (x', t') frame

$$\begin{pmatrix} L \\ t'_C \end{pmatrix} = \gamma t_C \begin{pmatrix} \alpha - \beta \\ -\alpha\beta + 1 \end{pmatrix}$$

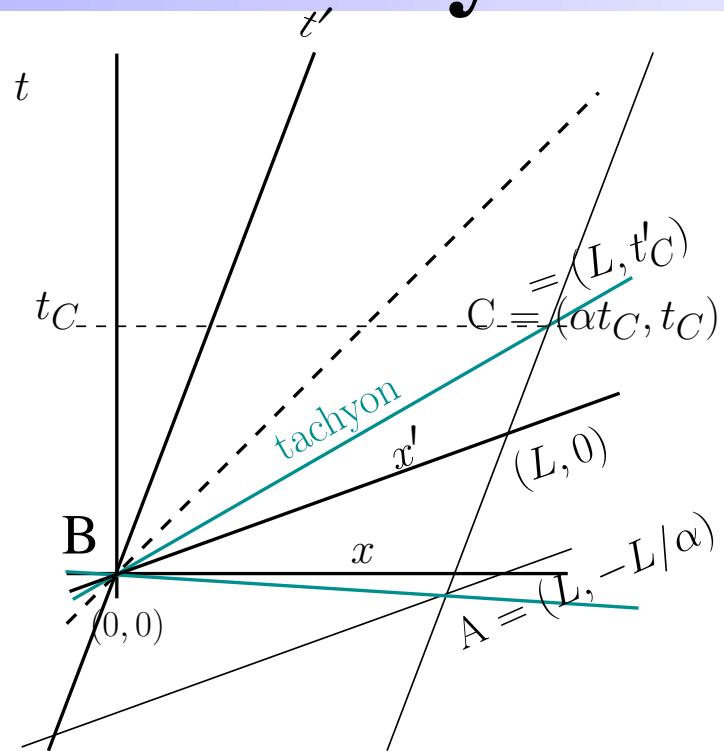
$$t'_C = L \frac{1 - \alpha\beta}{\alpha - \beta}$$

$$t'_C - t'_A = L \left(\frac{1 - \alpha\beta}{\alpha - \beta} + \frac{1}{\alpha} \right)$$





SR: tachyonic antitelephone



B stationary: (x, t)
frame
A moving at speed β :
 (x', t') frame

$$\begin{pmatrix} L \\ t'_C \end{pmatrix} = \gamma t_C \begin{pmatrix} \alpha - \beta \\ -\alpha\beta + 1 \end{pmatrix}$$

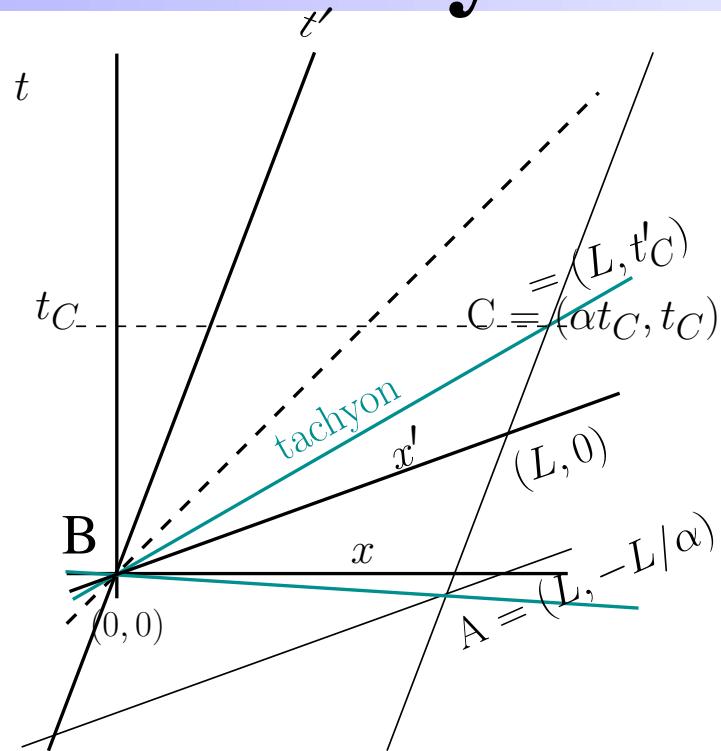
$$t'_C = L \frac{1 - \alpha\beta}{\alpha - \beta}$$

$$t'_C - t'_A = L \frac{\alpha - \alpha^2\beta + \alpha - \beta}{\alpha(\alpha - \beta)}$$





SR: tachyonic antitelephone



B stationary: (x, t)
frame
A moving at speed β :
 (x', t') frame

$$\begin{pmatrix} L \\ t'_C \end{pmatrix} = \gamma t_C \begin{pmatrix} \alpha - \beta \\ -\alpha\beta + 1 \end{pmatrix}$$

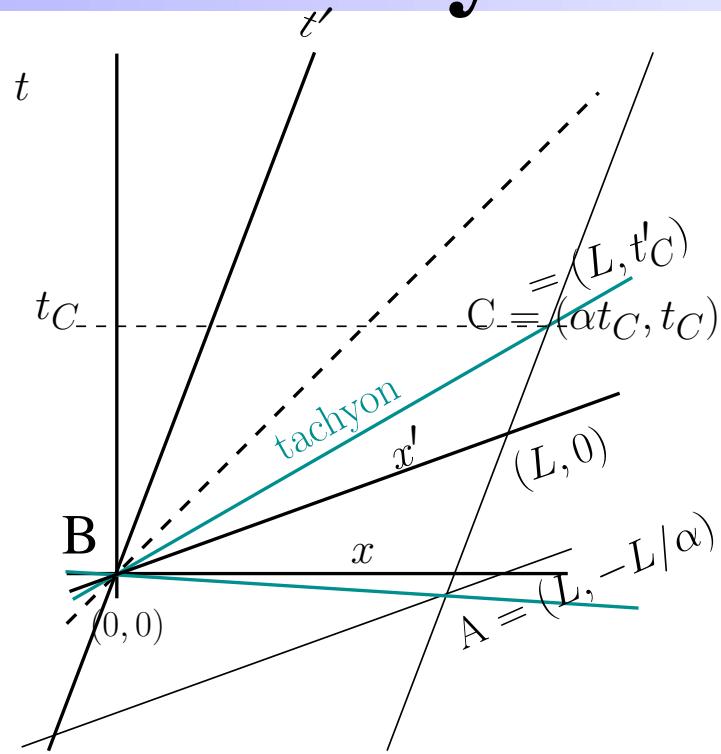
$$t'_C = L \frac{1 - \alpha\beta}{\alpha - \beta}$$

$$t'_C - t'_A = L \frac{2\alpha - (\alpha^2 + 1)\beta}{\alpha(\alpha - \beta)}$$





SR: tachyonic antitelephone



B stationary: (x, t)
frame

A moving at speed β :
 (x', t') frame

$$\begin{pmatrix} L \\ t'_C \end{pmatrix} = \gamma t_C \begin{pmatrix} \alpha - \beta \\ -\alpha\beta + 1 \end{pmatrix}$$

$$t'_C = L \frac{1 - \alpha\beta}{\alpha - \beta}$$

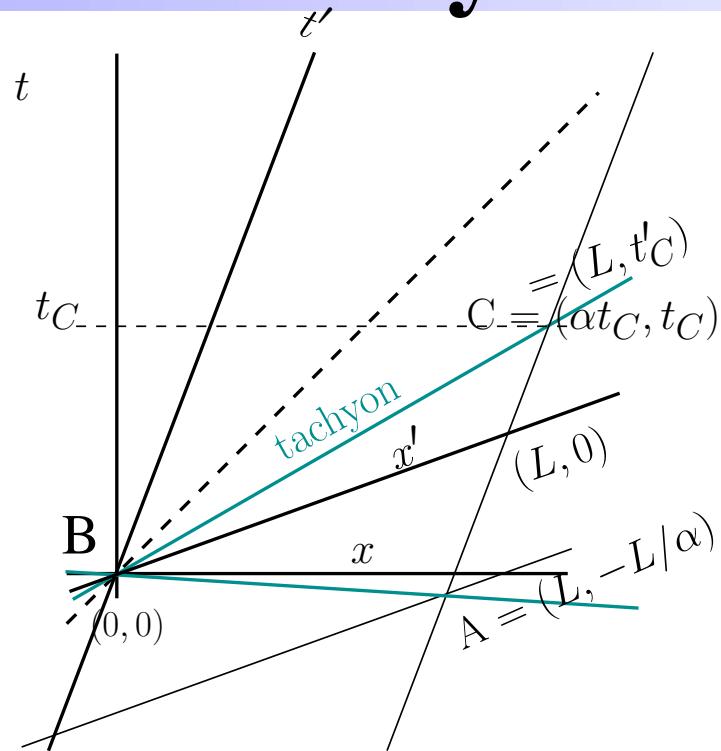
$$t'_C - t'_A = L \frac{2\alpha - (\alpha^2 + 1)\beta}{\alpha(\alpha - \beta)}$$

$$< 0 \text{ if } \beta > \frac{2\alpha}{\alpha^2 + 1}$$





SR: tachyonic antitelephone



B stationary: (x, t)
 frame
 A moving at speed β :
 (x', t') frame

$$\begin{pmatrix} L \\ t'_C \end{pmatrix} = \gamma t_C \begin{pmatrix} \alpha - \beta \\ -\alpha\beta + 1 \end{pmatrix}$$

$$t'_C = L \frac{1 - \alpha\beta}{\alpha - \beta}$$

$$t'_C - t'_A = L \frac{2\alpha - (\alpha^2 + 1)\beta}{\alpha(\alpha - \beta)}$$

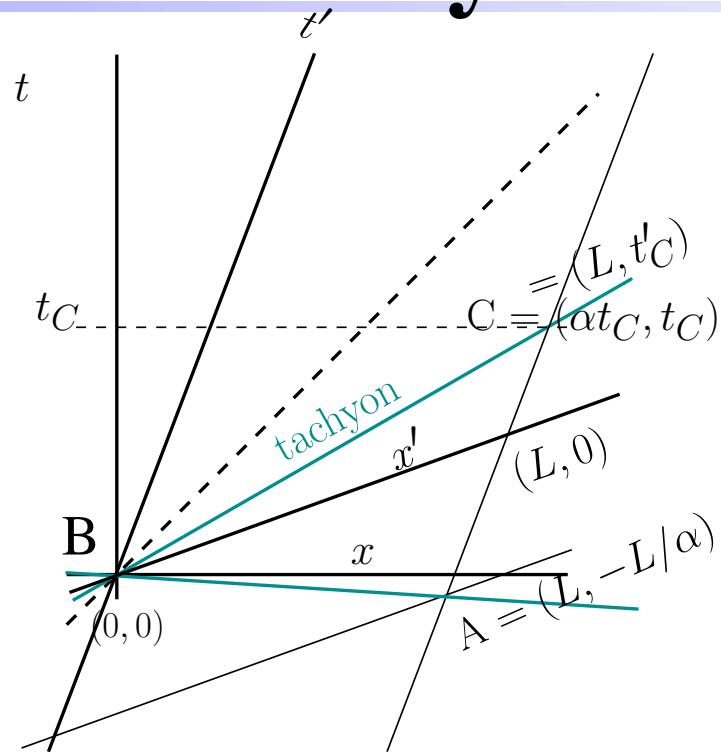
$$< 0 \text{ if } \beta > \frac{2\alpha}{\alpha^2 + 1}$$

A receives tachyonic response at C before sending it





SR: tachyonic antitelephone



B stationary: (x, t)
frame
A moving at speed β :
 (x', t') frame

$$\begin{pmatrix} L \\ t'_C \end{pmatrix} = \gamma t_C \begin{pmatrix} \alpha - \beta \\ -\alpha\beta + 1 \end{pmatrix}$$

$$t'_C = L \frac{1 - \alpha\beta}{\alpha - \beta}$$

$$t'_C - t'_A = L \frac{2\alpha - (\alpha^2 + 1)\beta}{\alpha(\alpha - \beta)}$$

$$< 0 \text{ if } \beta > \frac{2\alpha}{\alpha^2 + 1}$$

A receives tachyonic response at C before sending it

w:tachyonic antitelephone



SR: pole-barn/ladder paradox





SR: pole-barn/ladder paradox

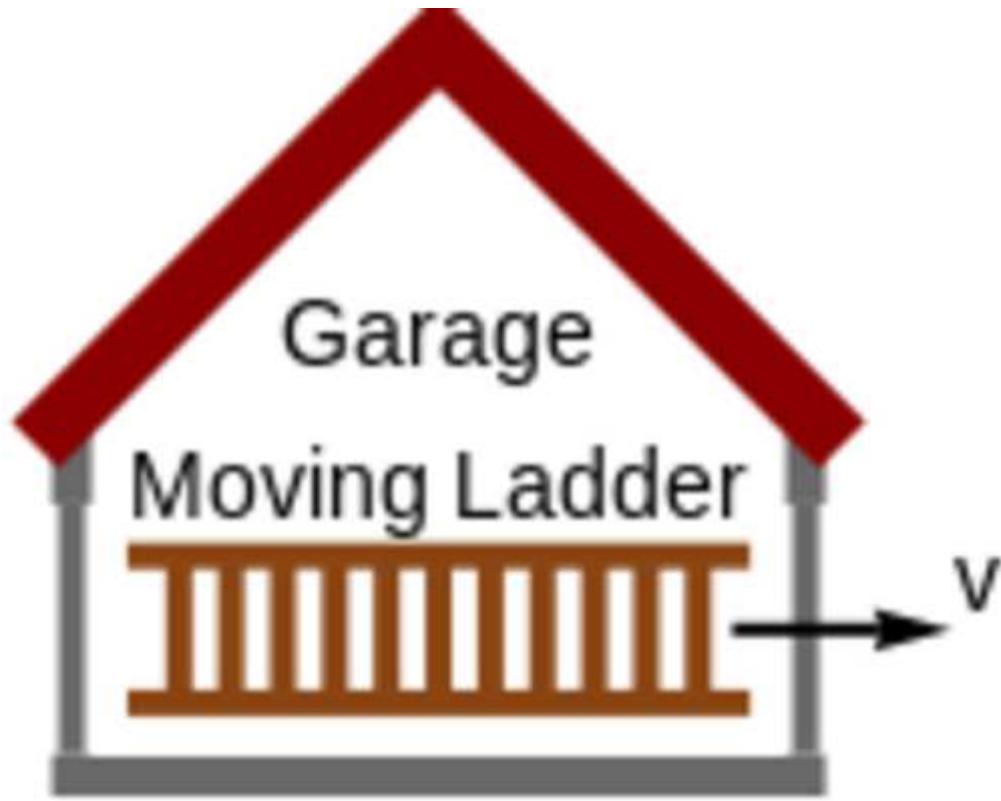


- ladder of length 29.9γ ns, garage length 30 ns





SR: pole-barn/ladder paradox

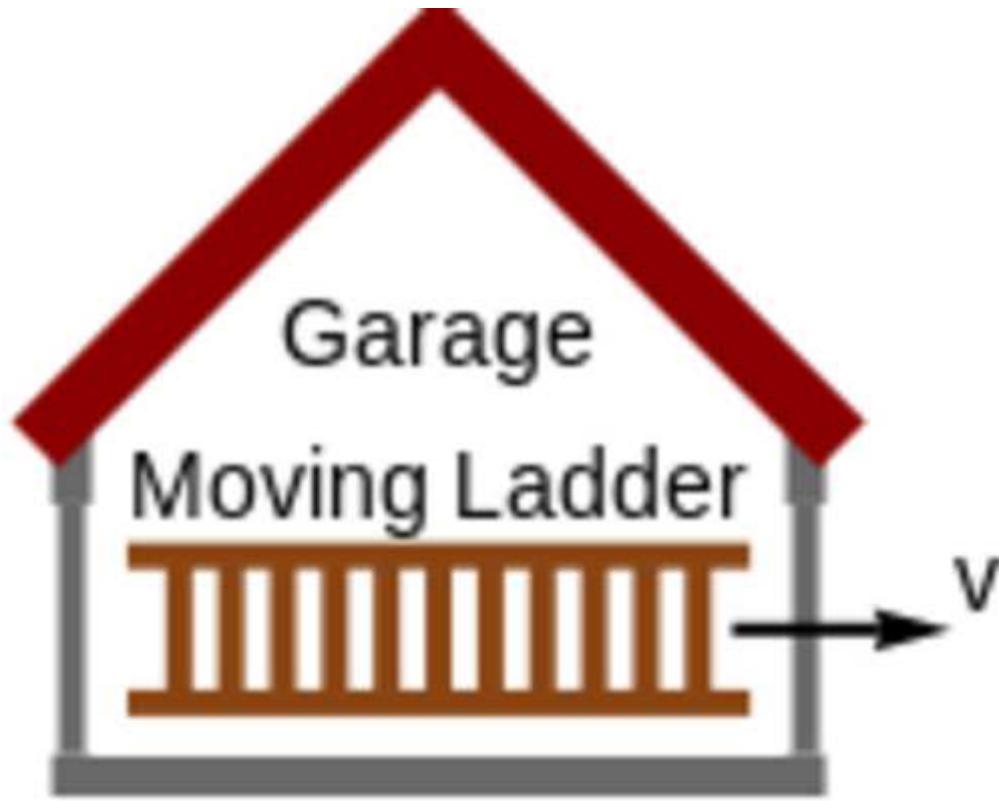


- ladder of length 29.9γ ns, garage length 30 ns
- instantaneously close front + back doors





SR: pole-barn/ladder paradox

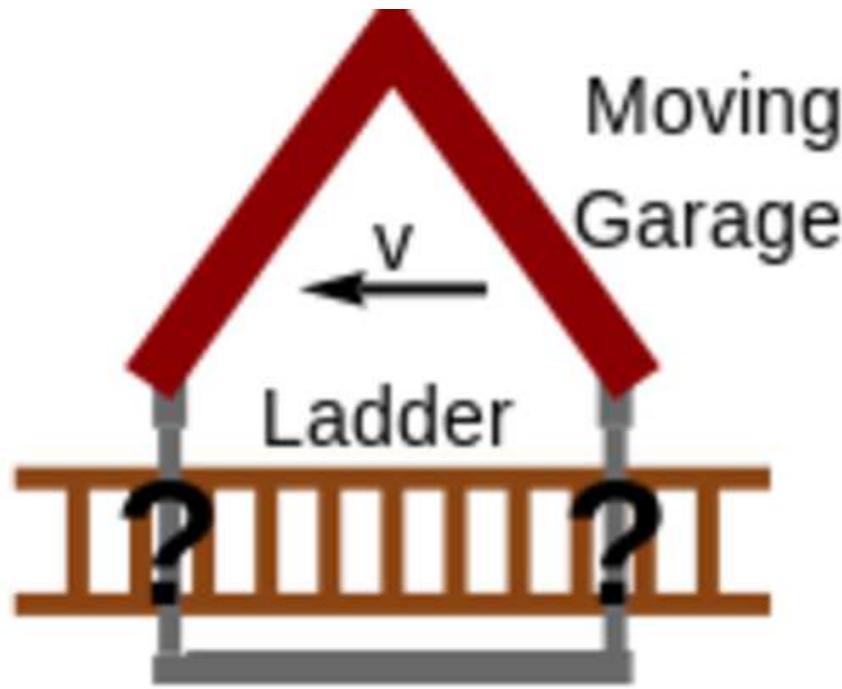


- ladder of length 29.9γ ns, garage length 30 ns
- instantaneously close front + back doors
- 29.9γ ns / $\gamma < 30$ ns \Rightarrow OK





SR: pole-barn/ladder paradox



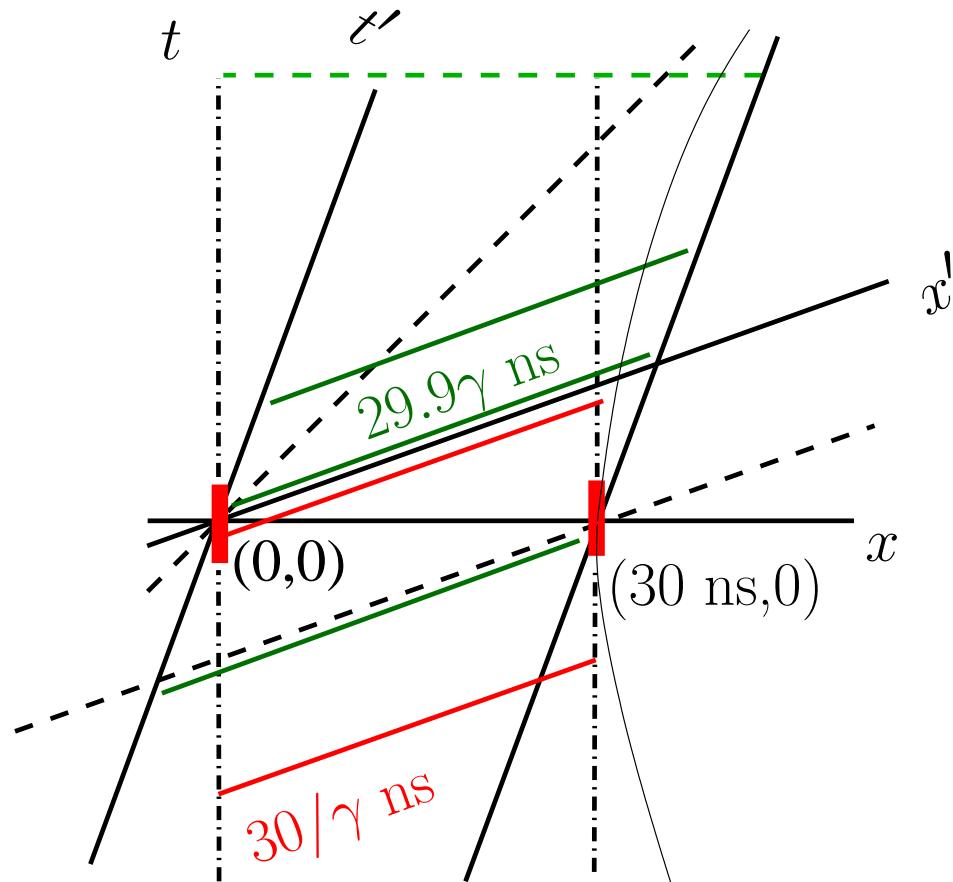
- ladder of length 29.9γ ns, garage length 30 ns
- instantaneously close front + back doors
- ladder frame: garage $30/\gamma$ ns long $\ll 29.9\gamma$ ns!!

Is this possible or not? Make a spacetime diagram.



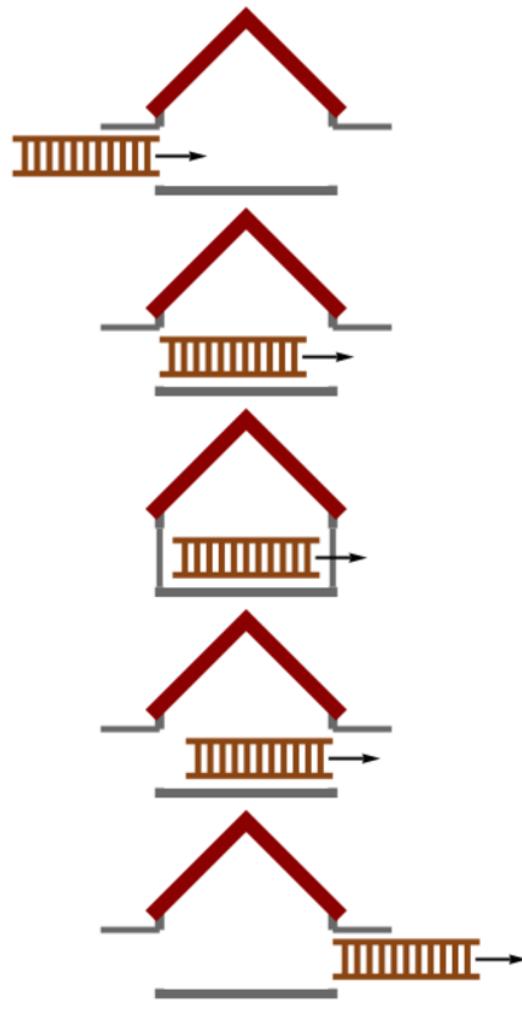


SR: pole-barn/ladder paradox



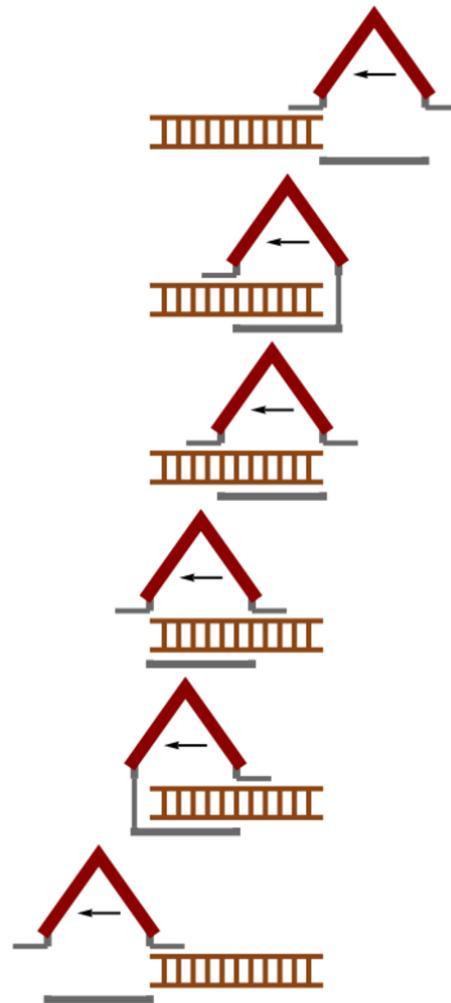


SR: pole-barn/ladder paradox



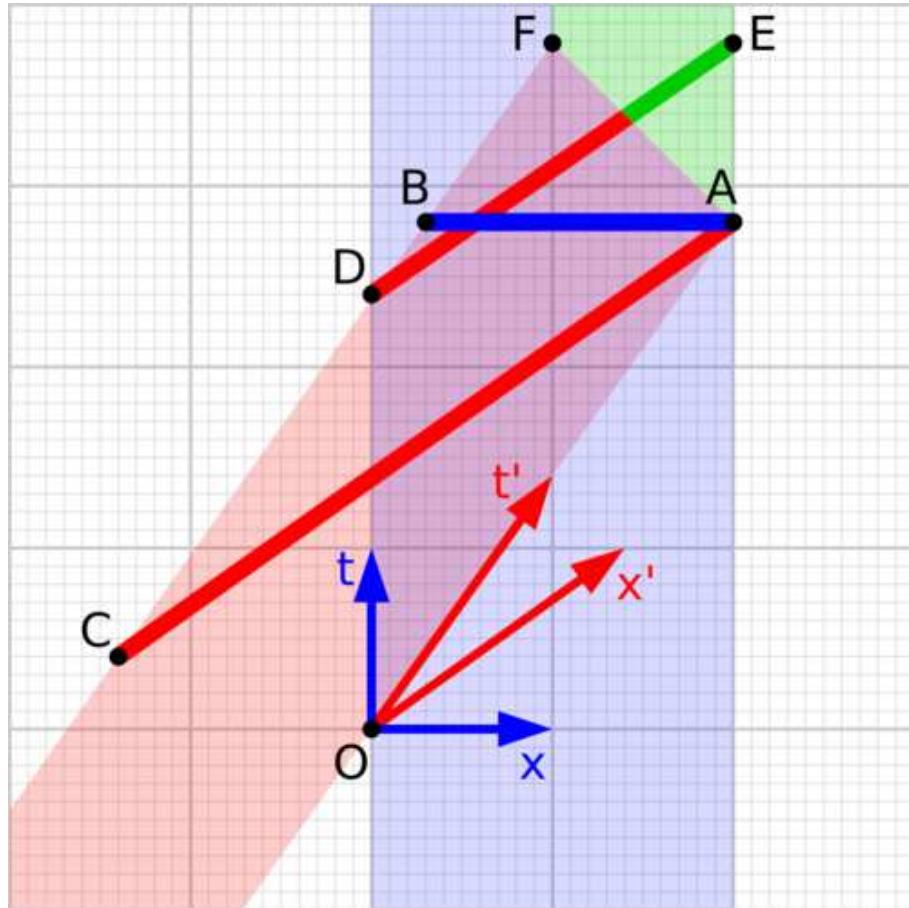


SR: pole-barn/ladder paradox





SR: pole-barn/ladder paradox

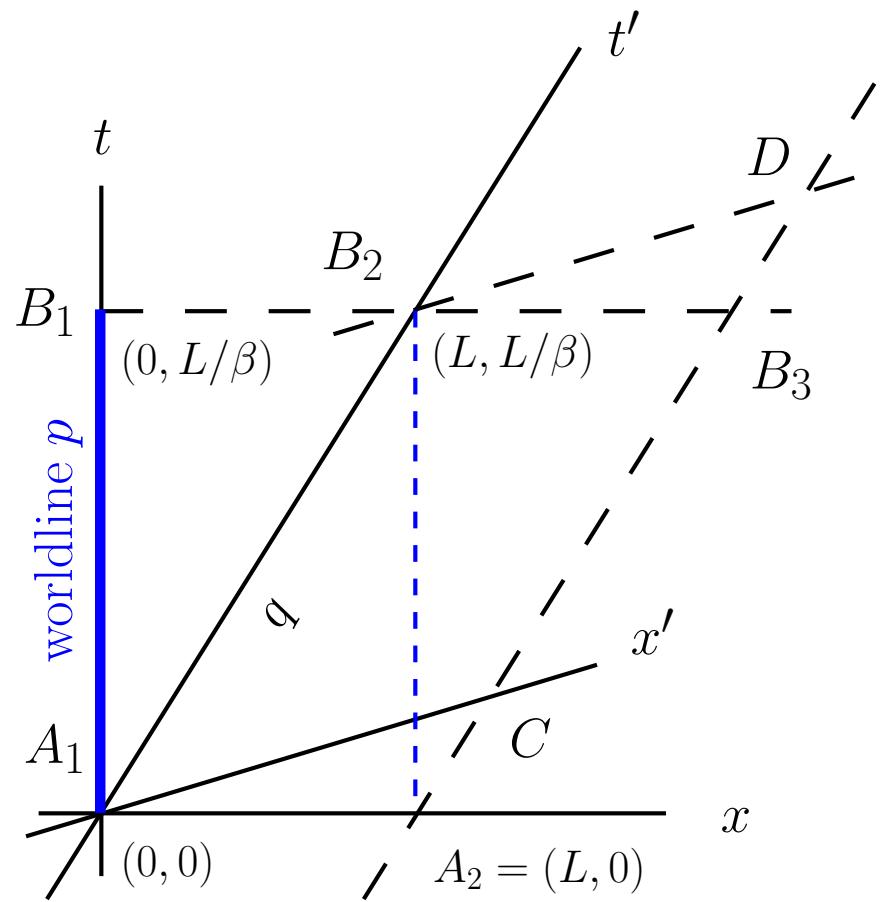


[w:Ladder paradox](#)





SR: twins paradox

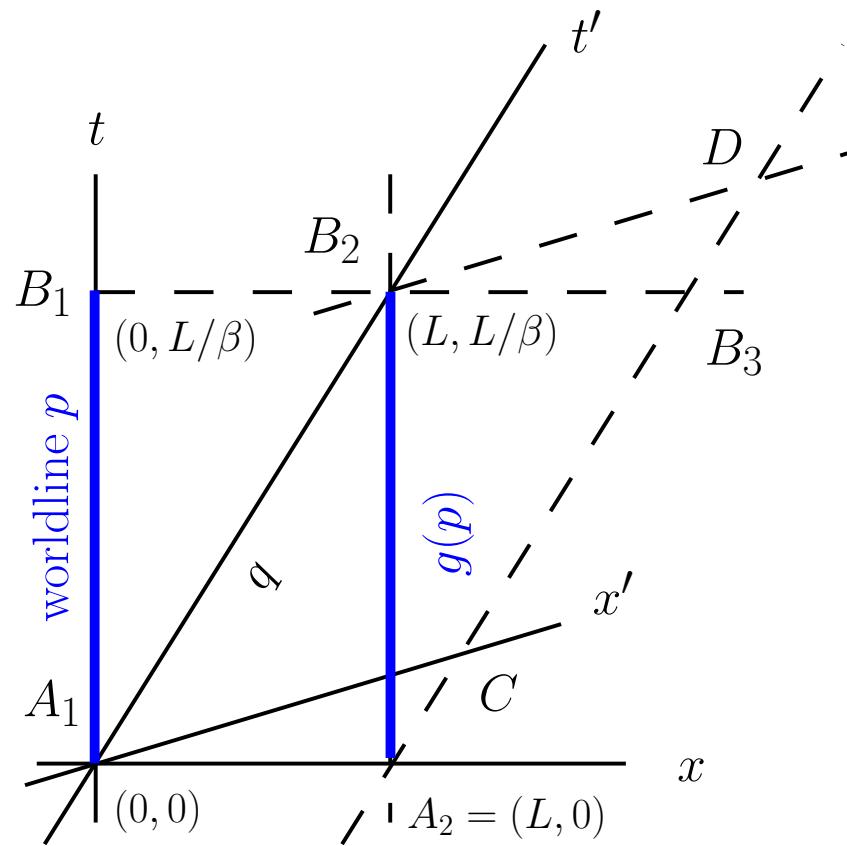


simply connected Minkowski





SR: twins paradox

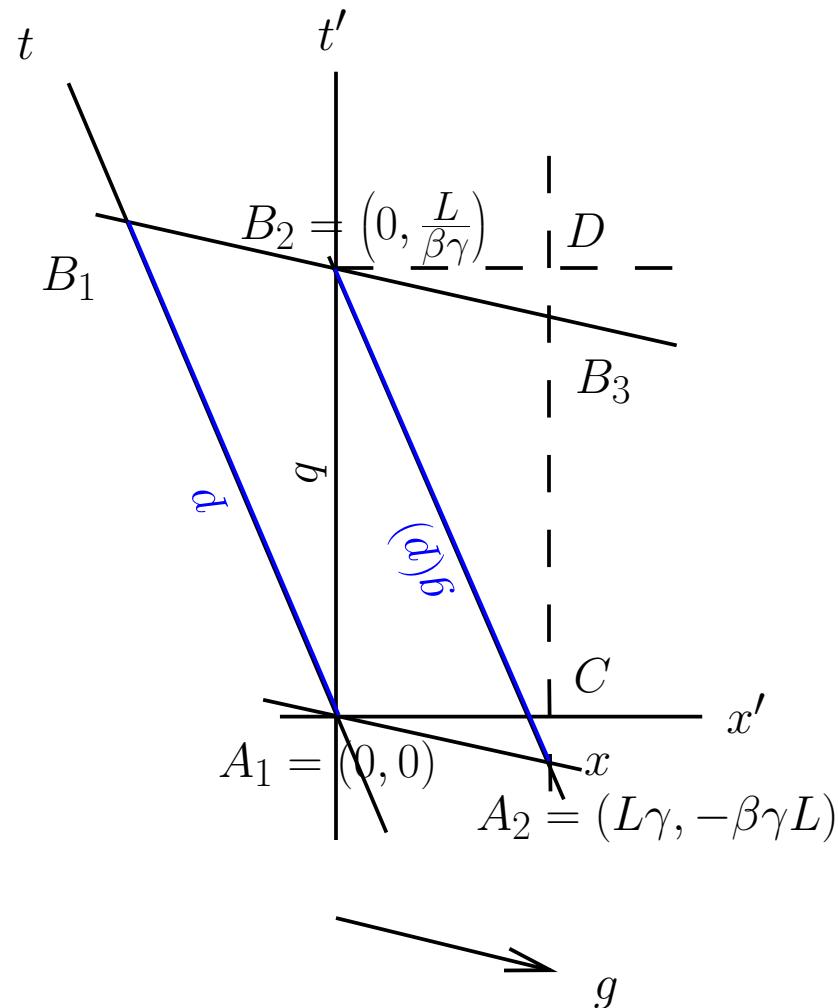


holonomy \overrightarrow{g}
identify spacetime events

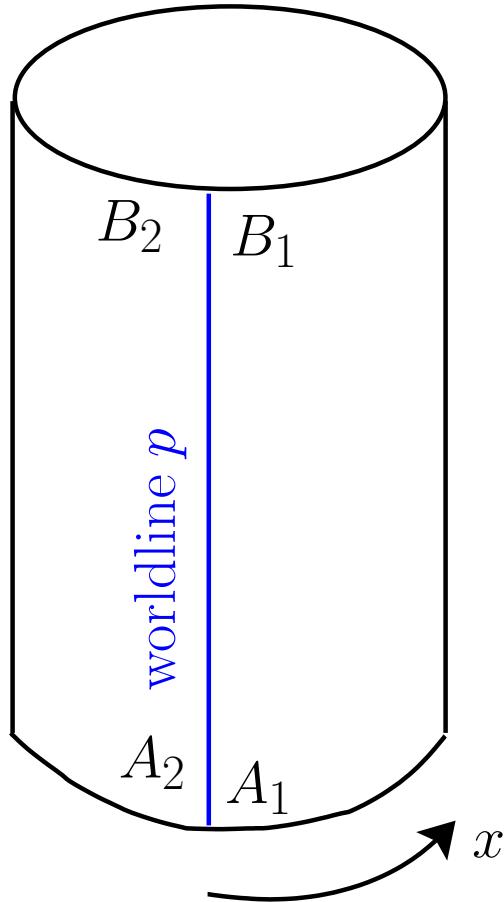




SR: twins paradox

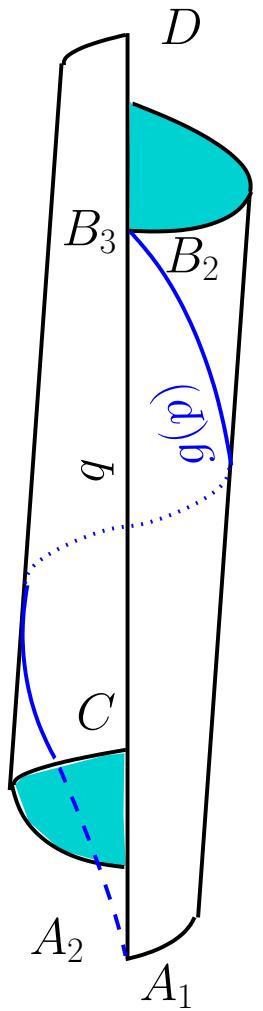


SR: twins paradox

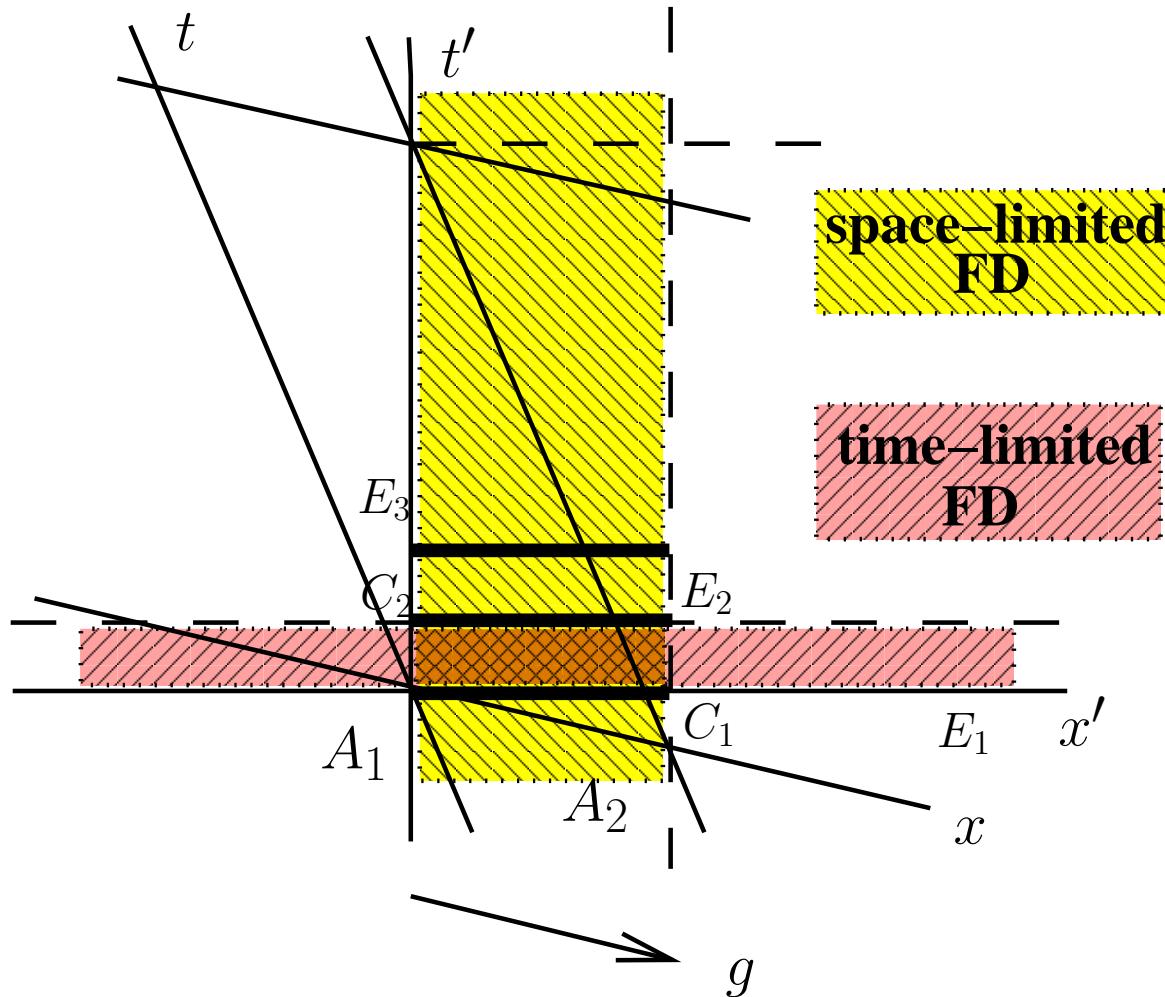




SR: twins paradox

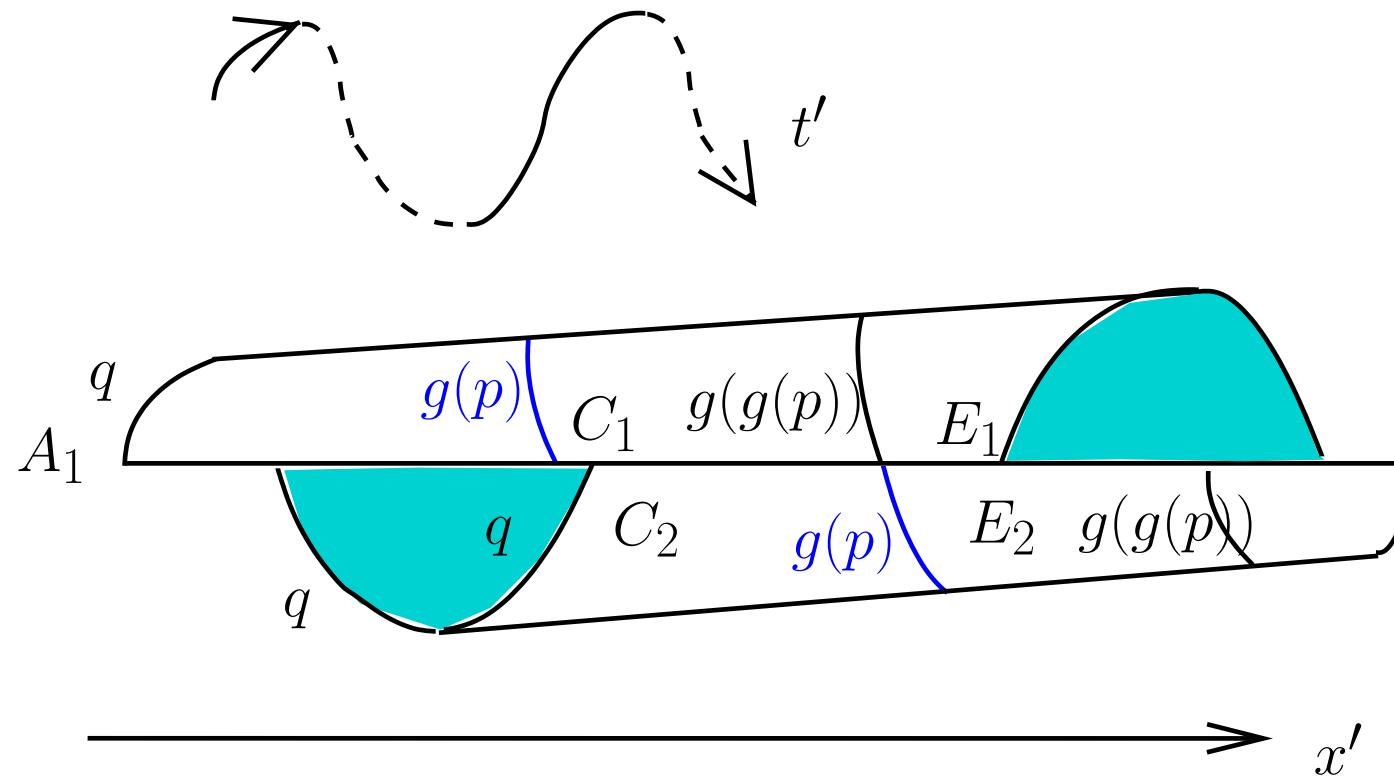


SR: twins paradox





SR: twins paradox



Roukema & Bajtlik 2008, MNRAS, 390, 655
[arXiv:astro-ph/0612155](https://arxiv.org/abs/astro-ph/0612155)

- helps understand w:Ehrenfest paradox



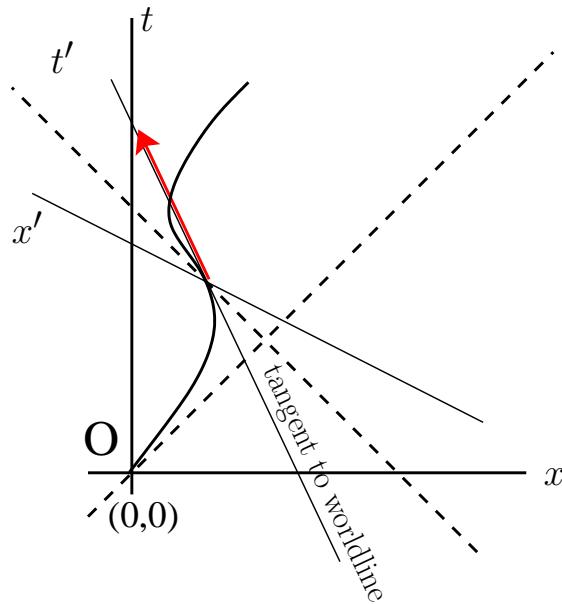
SR: four-velocity, four-momentum

choose x axis so that 3-velocity $u_{\text{Galilean}} = (\beta, 0, 0)^T$ for observer with $(t, x, y, z)^T$ coord system



SR: four-velocity, four-momentum

choose x axis so that 3-velocity $u_{\text{Galilean}} = (\beta, 0, 0)^T$ for observer with (t, x, y, z) coord system



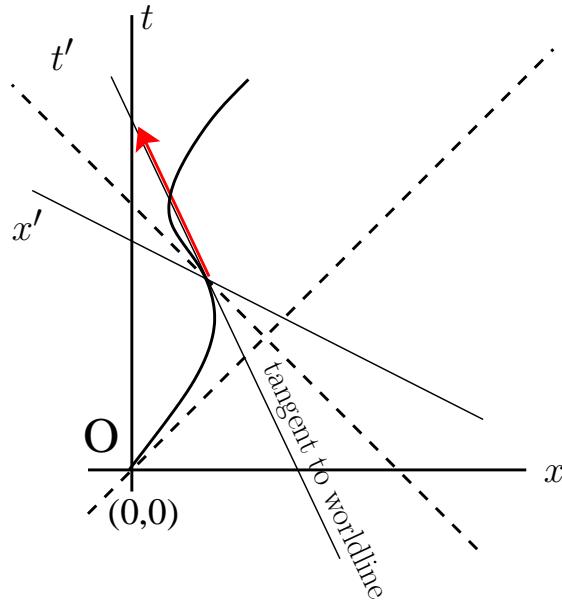
- in (t, x) spacetime 2-plane, extend from scalar speed β to spacetime vector = tangent to worldline

SR: four-velocity, four-momentum

choose x axis so that 3-velocity $u_{\text{Galilean}} = (\beta, 0, 0)^T$ for observer with (t, x, y, z) coord system

$$(u^t, u^x) := \left(\frac{d}{d\tau} t(\tau), \frac{d}{d\tau} x(\tau) \right)$$

w:four-velocity



- in (t, x) spacetime 2-plane, extend from scalar speed β to spacetime vector =
- L** tangent to worldline

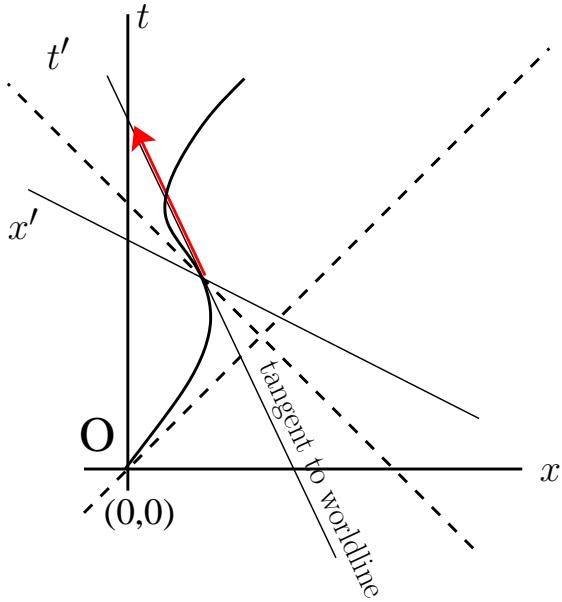
SR: four-velocity, four-momentum

choose x axis so that 3-velocity $u_{\text{Galilean}} = (\beta, 0, 0)^T$ for observer with (t, x, y, z) coord system

$$(u^t, u^x) := \left(\frac{d}{d\tau} t(\tau), \frac{d}{d\tau} x(\tau) \right)$$

w:four-velocity

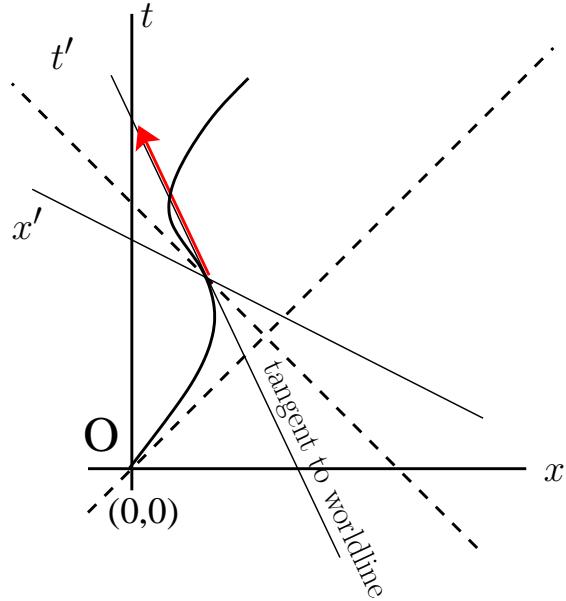
similarly $(u^{t'}, u^{x'}) = \left(\frac{d}{d\tau} t'(\tau), \frac{d}{d\tau} x'(\tau) \right)$



in (t, x) spacetime
2-plane, extend from

SR: four-velocity, four-momentum

choose x axis so that 3-velocity $u_{\text{Galilean}} = (\beta, 0, 0)^T$ for observer with (t, x, y, z) coord system



$$(u^t, u^x) := \left(\frac{d}{d\tau} t(\tau), \frac{d}{d\tau} x(\tau) \right)$$

w:**four-velocity**

similarly $(u^{t'}, u^{x'}) = \left(\frac{d}{d\tau} t'(\tau), \frac{d}{d\tau} x'(\tau) \right) = (1, 0)$

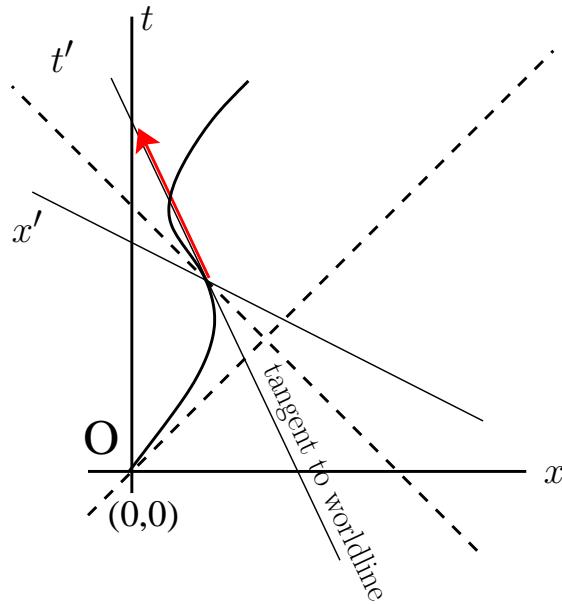
- in (t, x) spacetime 2-plane, extend from scalar speed β to spacetime vector =
- L**tangent to worldline

SR: four-velocity, four-momentum

choose x axis so that 3-velocity $u_{\text{Galilean}} = (\beta, 0, 0)^T$ for observer with (t, x, y, z) coord system

$$(u^t, u^x) := \left(\frac{d}{d\tau} t(\tau), \frac{d}{d\tau} x(\tau) \right)$$

w:four-velocity



$$\text{similarly } (u^{t'}, u^{x'}) = \left(\frac{d}{d\tau} t'(\tau), \frac{d}{d\tau} x'(\tau) \right) = (1, 0)$$

want \vec{u} Lorentz invariant \Rightarrow
 $(u^t, u^x)^T = \Lambda^{-1}(1, 0)^T$

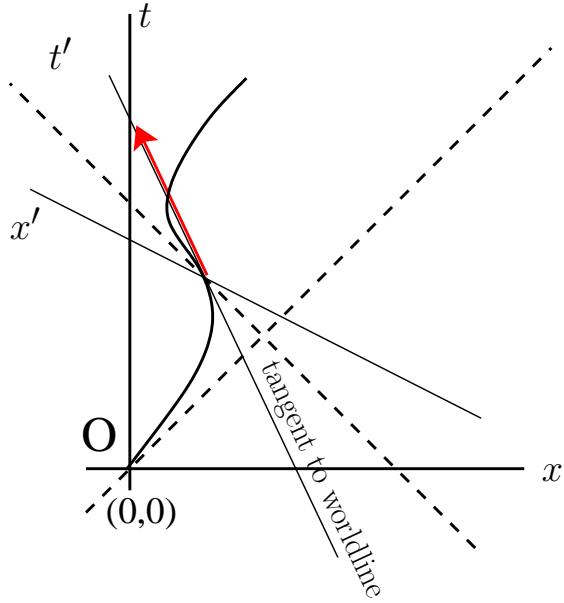
- in (t, x) spacetime

L²-plane, extend from

scalar speed β to
spacetime vector

SR: four-velocity, four-momentum

choose x axis so that 3-velocity $u_{\text{Galilean}} = (\beta, 0, 0)^T$ for observer with (t, x, y, z) coord system



- in (t, x) spacetime 2-plane, extend from scalar speed β to spacetime vector =
- L** tangent to worldline

$$(u^t, u^x) := \left(\frac{d}{d\tau} t(\tau), \frac{d}{d\tau} x(\tau) \right)$$

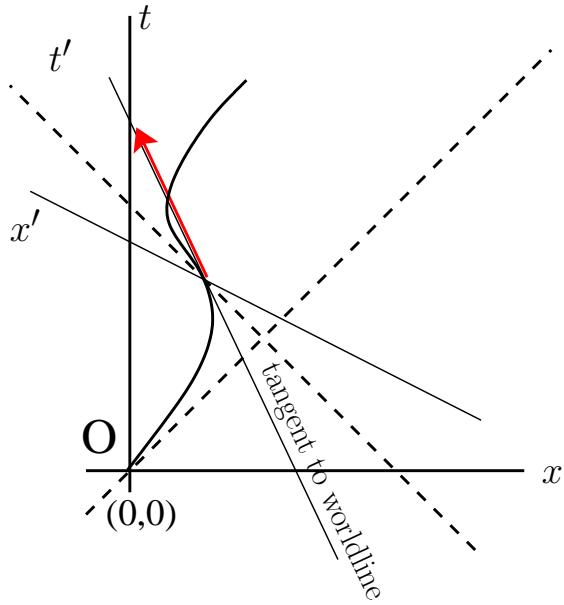
w:four-velocity

$$\text{similarly } (u^{t'}, u^{x'}) = \left(\frac{d}{d\tau} t'(\tau), \frac{d}{d\tau} x'(\tau) \right) = (1, 0)$$

want \vec{u} Lorentz invariant
 $\Rightarrow (u^t, u^x)^T = \Lambda^{-1}(1, 0)^T = \gamma(1, \beta)^T$

SR: four-velocity, four-momentum

choose x axis so that 3-velocity $u_{\text{Galilean}} = (\beta, 0, 0)^T$ for observer with (t, x, y, z) coord system



$$(u^t, u^x) := \left(\frac{d}{d\tau} t(\tau), \frac{d}{d\tau} x(\tau) \right)$$

w:four-velocity

$$\text{similarly } (u^{t'}, u^{x'}) = \left(\frac{d}{d\tau} t'(\tau), \frac{d}{d\tau} x'(\tau) \right) = (1, 0)$$

want \vec{u} Lorentz invariant
 $\Rightarrow (u^t, u^x)^T = \Lambda^{-1}(1, 0)^T = \gamma(1, \beta)^T$

4D: $\vec{u} = \gamma(1, \beta^x, \beta^y, \beta^z)^T$

notation in this pdf:

\vec{u} = 4-vector, ${}^{(3)}\vec{u}$ = spatial component

- in (t, x) spacetime 2-plane, extend from scalar speed β to spacetime vector = tangent to worldline

SR: four-velocity, four-momentum

Is the $(^3)$ -component (spatial component) of \vec{u} the same as the non-relativistic velocity?



SR: four-velocity, four-momentum

Is the $(^3)$ -component (spatial component) of \vec{u} the same as the non-relativistic velocity?

$$(^3)\vec{u} = \frac{d}{d\tau}(x, y, z)^T$$



SR: four-velocity, four-momentum



Is the $(^3)$ -component (spatial component) of \vec{u} the same as the non-relativistic velocity?

$$(^3)\vec{u} = \frac{d}{d\tau}(x, y, z)^T$$

$$= \gamma \frac{d}{dt}(x, y, z)^T$$



SR: four-velocity, four-momentum



Is the $(^3)$ -component (spatial component) of \vec{u} the same as the non-relativistic velocity?

$$(^3)\vec{u} = \frac{d}{d\tau}(x, y, z)^T$$

$$= \gamma \frac{d}{dt}(x, y, z)^T$$

$$\neq \frac{d}{dt}(x, y, z)^T \text{ except if } \beta = 0 \Leftrightarrow \gamma = 1$$



SR: four-velocity, four-momentum

momentum: $\vec{p} := m\vec{u} = m\gamma(1, \beta^x, \beta^y, \beta^z)^T$, where $m = \text{constant}$ w:invariant mass

^x ... = tensor-style component notation, not powers



SR: four-velocity, four-momentum

momentum: $\vec{p} := m\vec{u} = m\gamma(1, \beta^x, \beta^y, \beta^z)^T$, where $m = \text{constant}$ w:invariant mass



SR: four-velocity, four-momentum

momentum: $\vec{p} := m\vec{u} = m\gamma(1, \beta^x, \beta^y, \beta^z)^T$, where $m = \text{constant}$ [w:invariant mass](#)

What does the time component of momentum = $p^0 = m\gamma$ mean physically?

- first look at spatial component in a given ref. frame



SR: four-velocity, four-momentum

momentum: $\vec{p} := m\vec{u} = m\gamma(1, \beta^x, \beta^y, \beta^z)^T$, where $m = \text{constant}$ w:invariant mass

$$(3) \vec{p} = m \frac{d}{d\tau}(x, y, z)^T$$



SR: four-velocity, four-momentum

momentum: $\vec{p} := m\vec{u} = m\gamma(1, \beta^x, \beta^y, \beta^z)^T$, where $m = \text{constant}$ w:invariant mass

$$(3) \vec{p} = m \frac{d}{d\tau}(x, y, z)^T$$

$$= m\gamma \frac{d}{dt}(x, y, z)^T$$



SR: four-velocity, four-momentum

momentum: $\vec{p} := m\vec{u} = m\gamma(1, \beta^x, \beta^y, \beta^z)^T$, where $m = \text{constant}$ w:invariant mass

$$\begin{aligned}(3) \vec{p} &= m \frac{d}{d\tau} (x, y, z)^T \\&= m\gamma \frac{d}{dt} (x, y, z)^T \\&\neq m \frac{d}{dt} (x, y, z)^T \text{ except if } \beta = 0 \Leftrightarrow \gamma = 1\end{aligned}$$



SR: four-velocity, four-momentum

let us define 4-acceleration, 4-force



SR: four-velocity, four-momentum

$$(u^t, u^x) := \left(\frac{d}{d\tau} t(\tau), \frac{d}{d\tau} x(\tau) \right)$$



SR: four-velocity, four-momentum

$$\vec{a} := \frac{d}{d\tau} \vec{u}$$



SR: four-velocity, four-momentum

$$\vec{a} := \frac{d}{d\tau} \vec{u}$$

$${}^{(3)}\vec{a} = \frac{d^2}{d\tau^2} {}^{(3)}\vec{x}$$



SR: four-velocity, four-momentum

$$\vec{a} := \frac{d}{d\tau} \vec{u}$$

$$^{(3)}\vec{a} = \frac{d^2}{d\tau^2}(x, y, z)^T$$



SR: four-velocity, four-momentum



$$\vec{a} := \frac{d}{d\tau} \vec{u}$$

$${}^{(4)}\vec{f} := m \ {}^{(4)}\vec{a} \quad \text{defn } \underline{\text{w:four-force}}$$



SR: four-velocity, four-momentum



$$\vec{a} := \frac{d}{d\tau} \vec{u}$$

$${}^{(4)}\vec{f} := m {}^{(4)}\vec{a} \quad \text{defn } \underline{\text{w:four-force}}$$

$$= m \frac{d}{d\tau} \vec{u}$$



SR: four-velocity, four-momentum



$$\vec{a} := \frac{d}{d\tau} \vec{u}$$

$${}^{(4)}\vec{f} := m \ {}^{(4)}\vec{a} \quad \text{defn } \underline{\text{w:four-force}}$$

$$= \frac{d}{d\tau} \vec{p}$$





SR: invariance of ${}^{(4)}u$, ${}^{(4)}a$, ${}^{(4)}f$

Euclidean norm: $\|\vec{x}\|^2 = \sum_\mu (x^\mu)^2$



SR: invariance of ${}^{(4)}u$, ${}^{(4)}a$, ${}^{(4)}f$

Minkowski pseudo-norm: $\|\vec{x}\|^2 = \sum_{\mu,\nu} \eta_{\mu\nu} x^\mu x^\nu$





SR: invariance of ${}^{(4)}u$, ${}^{(4)}a$, ${}^{(4)}f$

Minkowski pseudo-norm: $\|\vec{x}\|^2 = \eta_{\mu\nu}x^\mu x^\nu$

w:Einstein summation sum is implicit





SR: invariance of ${}^{(4)}u$, ${}^{(4)}a$, ${}^{(4)}f$

Minkowski pseudo-norm: $\|\vec{x}\|^2 = -x^0x^0 + \delta_{ij}x^i x^j$

$\delta_{ij} = 1$ if $i = j$, otherwise = 0; $i, j \in 1, 2, 3$

invariance: $\|\vec{x}\|^2$ = same in all reference frames

sign convention: $(-, +, +, +)$ or $(+, -, -, -)$ are common



SR: invariance of ${}^{(4)}u$, ${}^{(4)}a$, ${}^{(4)}f$



Minkowski pseudo-norm: $\|\vec{x}\|^2 = -x^0x^0 + \delta_{ij}x^i x^j$

$\delta_{ij} = 1$ if $i = j$, otherwise = 0; $i, j \in 1, 2, 3$

invariance: $\|\vec{x}\|^2$ = same in all reference frames

sign convention: $(-, +, +, +)$ or $(+, -, -, -)$ are common

non-rest frame: $\|\vec{u}\|^2 = -\gamma^2 + \gamma^2\beta^2$





SR: invariance of ${}^{(4)}u$, ${}^{(4)}a$, ${}^{(4)}f$

Minkowski pseudo-norm: $\|\vec{x}\|^2 = -x^0x^0 + \delta_{ij}x^i x^j$

$\delta_{ij} = 1$ if $i = j$, otherwise = 0; $i, j \in 1, 2, 3$

invariance: $\|\vec{x}\|^2$ = same in all reference frames

sign convention: $(-, +, +, +)$ or $(+, -, -, -)$ are common

non-rest frame: $\|\vec{u}\|^2 = -\gamma^2 + \gamma^2\beta^2 = -1$





SR: invariance of ${}^{(4)}u$, ${}^{(4)}a$, ${}^{(4)}f$

Minkowski pseudo-norm: $\|\vec{x}\|^2 = -x^0x^0 + \delta_{ij}x^i x^j$

$\delta_{ij} = 1$ if $i = j$, otherwise = 0; $i, j \in 1, 2, 3$

invariance: $\|\vec{x}\|^2$ = same in all reference frames

sign convention: $(-, +, +, +)$ or $(+, -, -, -)$ are common

non-rest frame: $\|\vec{u}\|^2 = -\gamma^2 + \gamma^2\beta^2 = -1$

rest frame: $\|\vec{u}\|^2 = -1^2 + 0^2 = -1$ invariant





SR: invariance of ${}^{(4)}u$, ${}^{(4)}a$, ${}^{(4)}f$

Minkowski pseudo-norm: $\|\vec{x}\|^2 = -x^0x^0 + \delta_{ij}x^i x^j$

$\delta_{ij} = 1$ if $i = j$, otherwise = 0; $i, j \in 1, 2, 3$

invariance: $\|\vec{x}\|^2$ = same in all reference frames

sign convention: $(-, +, +, +)$ or $(+, -, -, -)$ are common

non-rest frame: $\|\vec{u}\|^2 = -\gamma^2 + \gamma^2\beta^2 = -1$

rest frame: $\|\vec{u}\|^2 = -1^2 + 0^2 = -1$ invariant

similarly: $\|\vec{a}\|^2$, $\|\vec{f}\|^2$ invariant



SR: energy: varies with ref frame

Newtonian $K = (1/2)m\beta^2 = 0$ in rest frame



SR: energy: varies with ref frame



Newtonian $K = (1/2)m\beta^2 = 0$ in rest frame

4-force \vec{f} is invariant, but

3-force usually *defined* to be frame-dependent:

3-force := $\frac{d}{dt}^{(3)}\vec{p} \neq \frac{d}{d\tau}^{(3)}\vec{p}$



SR: energy: varies with ref frame



Newtonian $K = (1/2)m\beta^2 = 0$ in rest frame

4-force \vec{f} is invariant, but

3-force usually *defined* to be frame-dependent:

$$\text{3-force} := \frac{d}{dt} {}^{(3)}\vec{p} \neq \frac{d}{d\tau} {}^{(3)}\vec{p}$$

$$\frac{d}{dt} {}^{(3)}\vec{p} = \frac{{}^{(3)}\vec{f}}{\gamma}$$



SR: energy: varies with ref frame



in (x, t) frame,

$K = \text{work done}$



SR: energy: varies with ref frame

in (x, t) frame,
 $K = \text{work done}$

$$= \int_0^{\beta_2} \frac{(3)\vec{f}}{\gamma} \cdot d\vec{x}$$



SR: energy: varies with ref frame

in (x, t) frame,
 $K = \text{work done}$

$$= \int_0^{\beta_2} \frac{(3)\vec{f}}{\gamma} \cdot d\vec{x}$$
$$= \int_0^{\beta_2} \frac{d}{dt} (3)\vec{p} \cdot d\vec{x}$$



SR: energy: varies with ref frame

in (x, t) frame,
 $K = \text{work done}$

$$\begin{aligned} &= \int_0^{\beta_2} \frac{(3)\vec{f}}{\gamma} \cdot d\vec{x} \\ &= \int_0^{\beta_2} \frac{d}{dt}(m\beta\gamma)dx \end{aligned}$$

(assume $(3)\vec{f}/\gamma \parallel \vec{x}$)



SR: energy: varies with ref frame

in (x, t) frame,

$$= \int_0^{\beta_2} \frac{(3)\vec{f}}{\gamma} \cdot d\vec{x}$$

$K = \text{work done}$

$$= \int_0^{\beta_2} \frac{d}{dt}(m\beta\gamma)dx = m \int_0^{\beta_2} d(\beta\gamma) \frac{dx}{dt}$$



SR: energy: varies with ref frame

$$\begin{aligned} \text{in } (x, t) \text{ frame, } &= \int_0^{\beta_2} \frac{(3)\vec{f}}{\gamma} \cdot d\vec{x} \\ K = \text{work done} &= \int_0^{\beta_2} \frac{d}{dt}(m\beta\gamma)dx = m \int_0^{\beta_2} d(\beta\gamma)\beta \end{aligned}$$



SR: energy: varies with ref frame

$$\begin{aligned} \text{in } (x, t) \text{ frame, } &= \int_0^{\beta_2} \frac{(3)\vec{f}}{\gamma} \cdot d\vec{x} \\ K = \text{work done} &= \int_0^{\beta_2} \frac{d}{dt}(m\beta\gamma)dx = m \int_0^{\beta_2} d(\beta\gamma)\beta \\ &= m \int_0^{\beta_2} \beta(\beta d\gamma + \gamma d\beta) \end{aligned}$$



SR: energy: varies with ref frame

in (x, t) frame,

$$K = \text{work done} = \int_0^{\beta_2} \frac{(3)\vec{f}}{\gamma} \cdot d\vec{x}$$
$$= \int_0^{\beta_2} \frac{d}{dt}(m\beta\gamma)dx = m \int_0^{\beta_2} d(\beta\gamma)\beta$$
$$= m \int_0^{\beta_2} \beta(\beta d\gamma + \gamma d\beta)$$
$$= m \int_{\gamma=1}^{\gamma=\gamma_2} (\beta^2 + \gamma^{-2})d\gamma \Leftarrow d\gamma = \beta\gamma^3d\beta$$



SR: energy: varies with ref frame

in (x, t) frame,

$$K = \text{work done} = \int_0^{\beta_2} \frac{(3)\vec{f}}{\gamma} \cdot d\vec{x}$$
$$= \int_0^{\beta_2} \frac{d}{dt}(m\beta\gamma)dx = m \int_0^{\beta_2} d(\beta\gamma)\beta$$
$$= m \int_0^{\beta_2} \beta(\beta d\gamma + \gamma d\beta)$$
$$= m \int_{\gamma=1}^{\gamma=\gamma_2} (\beta^2 + \gamma^{-2})d\gamma \Leftarrow d\gamma = \beta\gamma^3d\beta$$
$$= m \int_{\gamma=1}^{\gamma=\gamma_2} [\beta^2 + (1 - \beta^2)]d\gamma$$



SR: energy: varies with ref frame

$$\begin{aligned} \text{in } (x, t) \text{ frame, } &= \int_0^{\beta_2} \frac{(3)\vec{f}}{\gamma} \cdot d\vec{x} \\ K = \text{work done} &= \int_0^{\beta_2} \frac{d}{dt}(m\beta\gamma)dx = m \int_0^{\beta_2} d(\beta\gamma)\beta \\ &= m \int_0^{\beta_2} \beta(\beta d\gamma + \gamma d\beta) \\ &= m \int_{\gamma=1}^{\gamma=\gamma_2} (\beta^2 + \gamma^{-2})d\gamma \Leftarrow d\gamma = \beta\gamma^3d\beta \\ &= m \int_{\gamma=1}^{\gamma=\gamma_2} d\gamma \end{aligned}$$



SR: energy: varies with ref frame

in (x, t) frame,

$$K = \text{work done} = \int_0^{\beta_2} \frac{(3)\vec{f}}{\gamma} \cdot d\vec{x}$$
$$= \int_0^{\beta_2} \frac{d}{dt}(m\beta\gamma)dx = m \int_0^{\beta_2} d(\beta\gamma)\beta$$
$$= m \int_0^{\beta_2} \beta(\beta d\gamma + \gamma d\beta)$$
$$= m \int_{\gamma=1}^{\gamma=\gamma_2} (\beta^2 + \gamma^{-2})d\gamma \Leftarrow d\gamma = \beta\gamma^3d\beta$$
$$= m \int_{\gamma=1}^{\gamma=\gamma_2} d\gamma = m\gamma_2 - m$$



SR: energy: varies with ref frame

$$\begin{aligned} \text{in } (x, t) \text{ frame, } \quad &= \int_0^{\beta_2} \frac{(3)\vec{f}}{\gamma} \cdot d\vec{x} \\ K = \text{work done} \quad &= \int_0^{\beta_2} \frac{d}{dt}(m\beta\gamma)dx = m \int_0^{\beta_2} d(\beta\gamma)\beta \\ &= m \int_0^{\beta_2} \beta(\beta d\gamma + \gamma d\beta) \\ &= m \int_{\gamma=1}^{\gamma=\gamma_2} (\beta^2 + \gamma^{-2})d\gamma \Leftarrow d\gamma = \beta\gamma^3d\beta \\ &= m \int_{\gamma=1}^{\gamma=\gamma_2} d\gamma = m\gamma_2 - m \end{aligned}$$

$$\Rightarrow K + m = m\gamma \text{ drop "2"}$$



SR: energy: varies with ref frame

$$\begin{aligned} \text{in } (x, t) \text{ frame, } \quad &= \int_0^{\beta_2} \frac{(3)\vec{f}}{\gamma} \cdot d\vec{x} \\ K = \text{work done} \quad &= \int_0^{\beta_2} \frac{d}{dt}(m\beta\gamma)dx = m \int_0^{\beta_2} d(\beta\gamma)\beta \\ &= m \int_0^{\beta_2} \beta(\beta d\gamma + \gamma d\beta) \\ &= m \int_{\gamma=1}^{\gamma=\gamma_2} (\beta^2 + \gamma^{-2})d\gamma \Leftarrow d\gamma = \beta\gamma^3d\beta \\ &= m \int_{\gamma=1}^{\gamma=\gamma_2} d\gamma = m\gamma_2 - m \end{aligned}$$

$$\Rightarrow K + m = m\gamma = p^0$$



SR: energy: varies with ref frame

in (x, t) frame,

$$K = \text{work done} = \int_0^{\beta_2} \frac{(3)\vec{f}}{\gamma} \cdot d\vec{x}$$
$$= \int_0^{\beta_2} \frac{d}{dt}(m\beta\gamma)dx = m \int_0^{\beta_2} d(\beta\gamma)\beta$$
$$= m \int_0^{\beta_2} \beta(\beta d\gamma + \gamma d\beta)$$
$$= m \int_{\gamma=1}^{\gamma=\gamma_2} (\beta^2 + \gamma^{-2})d\gamma \Leftarrow d\gamma = \beta\gamma^3 d\beta$$
$$= m \int_{\gamma=1}^{\gamma=\gamma_2} d\gamma = m\gamma_2 - m$$

$$\Rightarrow K + m = m\gamma = p^0$$

so p^0 = kinetic energy + rest mass



SR: energy: varies with ref frame

Does small β limit agree with Newtonian K ?



SR: energy: varies with ref frame



Does small β limit agree with Newtonian K ?

momentum time component:

$$\begin{aligned} p^0 &= m\gamma = m(1 - \beta^2)^{-1/2} \\ &= m[1 - (1/2)(-\beta^2) + \mathcal{O}(\beta^4)] \text{ if } \beta \ll 1 \end{aligned}$$



SR: energy: varies with ref frame



Does small β limit agree with Newtonian K ?

momentum time component:

$$p^0 = m\gamma = m(1 - \beta^2)^{-1/2}$$

$$\approx m[1 + (1/2)\beta^2] \text{ if } \beta \ll 1$$



SR: energy: varies with ref frame



Does small β limit agree with Newtonian K ?

momentum time component:

$$p^0 = m\gamma = m(1 - \beta^2)^{-1/2}$$

$$\approx m + (1/2)m\beta^2 \text{ if } \beta \ll 1$$

Yes.





SR: $\vec{p} \dots$: invariant or not?

momentum: $\vec{p} = m\gamma(1, \beta^x, \beta^y, \beta^z)^T$

$$p^0 = m + K = m\gamma$$





SR: \vec{p} ...: invariant or not?

momentum: $\vec{p} = m\gamma(1, \beta^x, \beta^y, \beta^z)^T$

$$p^0 = m + K = m\gamma$$

non-rest frame: $\|\vec{u}\|^2 = -\gamma^2 + \gamma^2\beta^2 = -1$

rest frame: $\|\vec{u}\|^2 = -1^2 + 0^2 = -1$ invariant





SR: \vec{p} ...: invariant or not?

momentum: $\vec{p} = m\gamma(1, \beta^x, \beta^y, \beta^z)^T$

$$p^0 = m + K = m\gamma$$

non-rest frame: $\|\vec{p}\|^2 = -m^2\gamma^2 + m^2\gamma^2\beta^2 = -m^2$

rest frame: $\|\vec{p}\|^2 = -m^2 + 0^2 = -m^2$ invariant





SR: \vec{p} . . . : invariant or not?

momentum: $\vec{p} = m\gamma(1, \beta^x, \beta^y, \beta^z)^T$

$$p^0 = m + K = m\gamma$$

non-rest frame: $\|\vec{p}\|^2 = -m^2\gamma^2 + m^2\gamma^2\beta^2 = -m^2$

rest frame: $\|\vec{p}\|^2 = -m^2 + 0^2 = -m^2$ invariant

m = w:invariant mass \equiv rest mass: invariant

AND conserved (in interactions): $\|\vec{p} + \vec{q}\|^2 = \|\vec{r}\|^2$





SR: \vec{p} ...: invariant or not?

momentum: $\vec{p} = m\gamma(1, \beta^x, \beta^y, \beta^z)^T$

$$p^0 = m + K = m\gamma$$

non-rest frame: $\|\vec{p}\|^2 = -m^2\gamma^2 + m^2\gamma^2\beta^2 = -m^2$

rest frame: $\|\vec{p}\|^2 = -m^2 + 0^2 = -m^2$ invariant

m = w:invariant mass \equiv rest mass: invariant

AND conserved (in interactions): $\|\vec{p} + \vec{q}\|^2 = \|\vec{r}\|^2$

where interaction is (4-momenta):

$$\vec{p} + \vec{q} \rightarrow \vec{r}$$





SR: \vec{p} . . . : invariant or not?

momentum: $\vec{p} = m\gamma(1, \beta^x, \beta^y, \beta^z)^T$

$$p^0 = m + K = m\gamma$$

non-rest frame: $\|\vec{p}\|^2 = -m^2\gamma^2 + m^2\gamma^2\beta^2 = -m^2$

rest frame: $\|\vec{p}\|^2 = -m^2 + 0^2 = -m^2$ invariant

m = w:invariant mass \equiv rest mass: invariant

AND conserved (in interactions): $\|\vec{p} + \vec{q}\|^2 = \|\vec{r}\|^2$

where interaction is (4-momenta):

$$\vec{p} + \vec{q} \rightarrow \vec{r}$$

WARNING: assume that 4-momentum vectors at different space-time positions can be parallel-transported; not the case in curved spacetime





SR: $\vec{p} \dots$: invariant or not?

vector space $\Rightarrow p^i + q^i = r^i$ ($i = 1, 2, 3$)





SR: $\vec{p} \dots$: invariant or not?

vector space $\Rightarrow p^i + q^i = r^i$ ($i = 1, 2, 3$)

= conservation of (relativistic) $^{(3)}$ -momentum (Newtonian:
conserved)

but NOT invariant (Newtonian: not invariant)





SR: $\vec{p} \dots$: invariant or not?

vector space $\Rightarrow p^i + q^i = r^i$ ($i = 1, 2, 3$)

= conservation of (relativistic) $^{(3)}$ -momentum (Newtonian: conserved)

but NOT invariant (Newtonian: not invariant)

vector space $\Rightarrow p^0 + q^0 = r^0$





SR: $\vec{p} \dots$: invariant or not?

vector space $\Rightarrow p^i + q^i = r^i$ ($i = 1, 2, 3$)

= conservation of (relativistic) $^{(3)}$ -momentum (Newtonian: conserved)

but NOT invariant (Newtonian: not invariant)

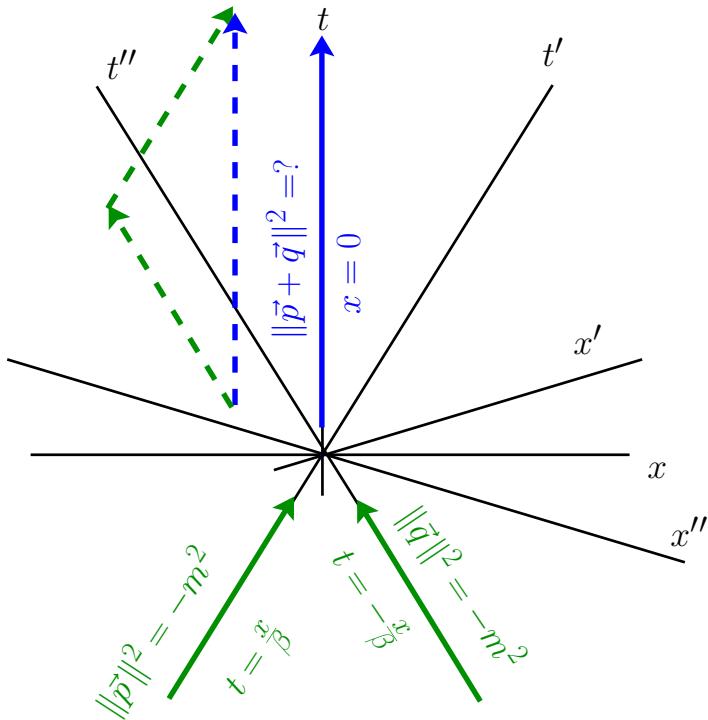
vector space $\Rightarrow p^0 + q^0 = r^0$

= conservation of (relativistic) "total energy" = $m + K$
(Newtonian: m conserved, K not conserved, $K +$ potential energy conserved)

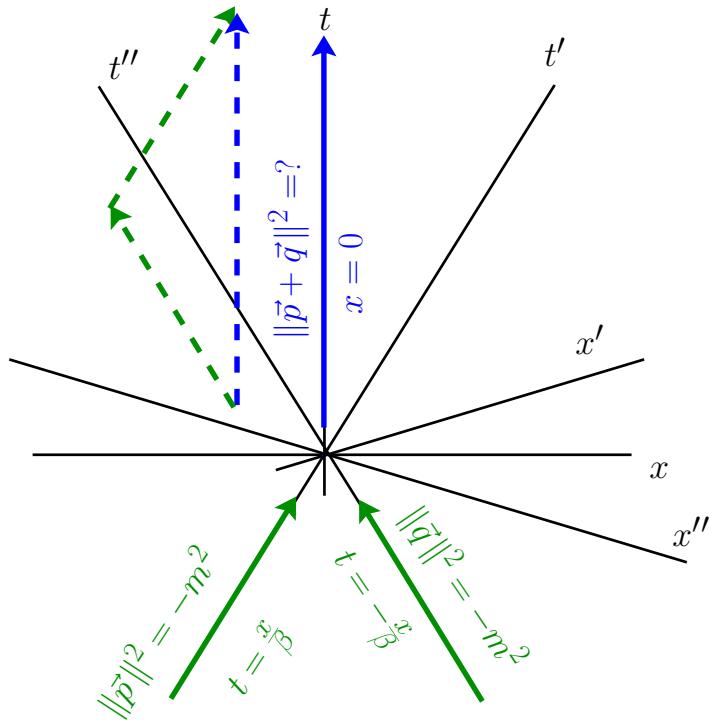
but NOT invariant (Newtonian: m invariant, K not invariant)



SR: $\vec{p} \dots$: invariant or not?



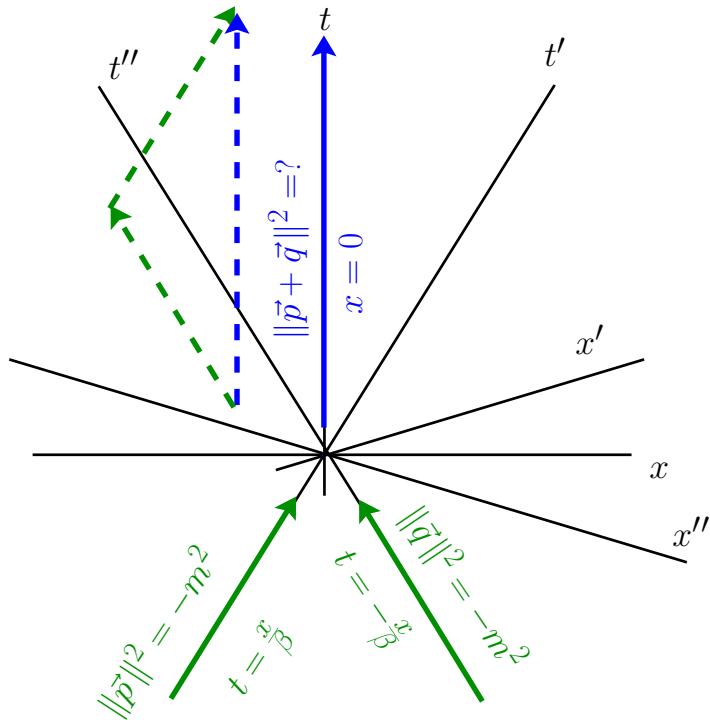
SR: $\vec{p} \dots$: invariant or not?



$$\vec{p} = m(\gamma, +\beta\gamma, 0, 0)^T$$



SR: $\vec{p} \dots$: invariant or not?

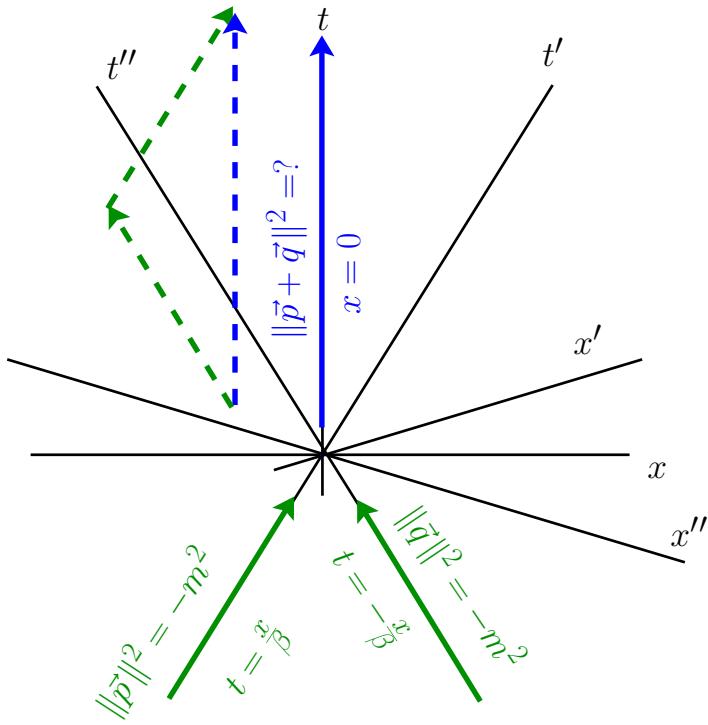


$$\vec{p} = m(\gamma, +\beta\gamma, 0, 0)^T$$

$$\vec{q} = m(\gamma, -\beta\gamma, 0, 0)^T$$



SR: $\vec{p} \dots$: invariant or not?



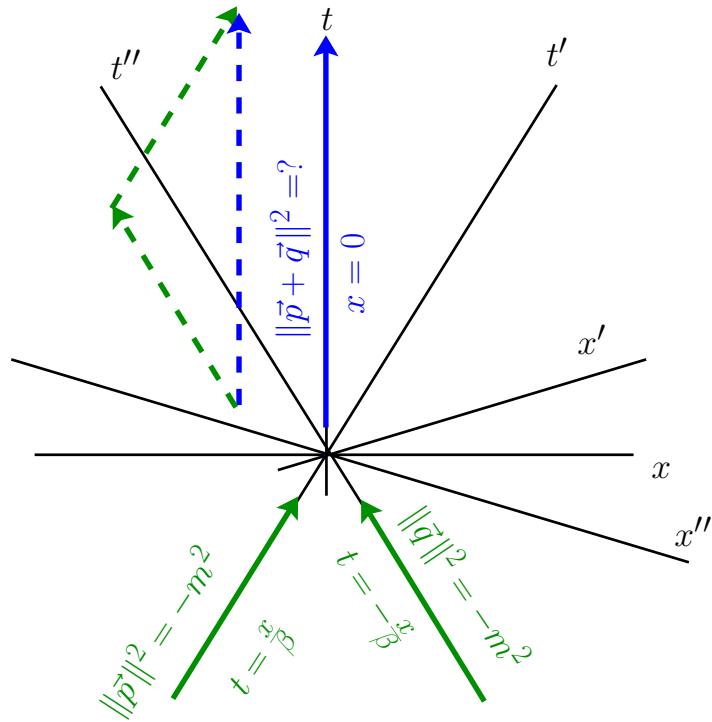
$$\vec{p} = m(\gamma, +\beta\gamma, 0, 0)^T$$

$$\vec{q} = m(\gamma, -\beta\gamma, 0, 0)^T$$

$$\vec{r} = \vec{p} + \vec{q}$$



SR: $\vec{p} \dots$: invariant or not?



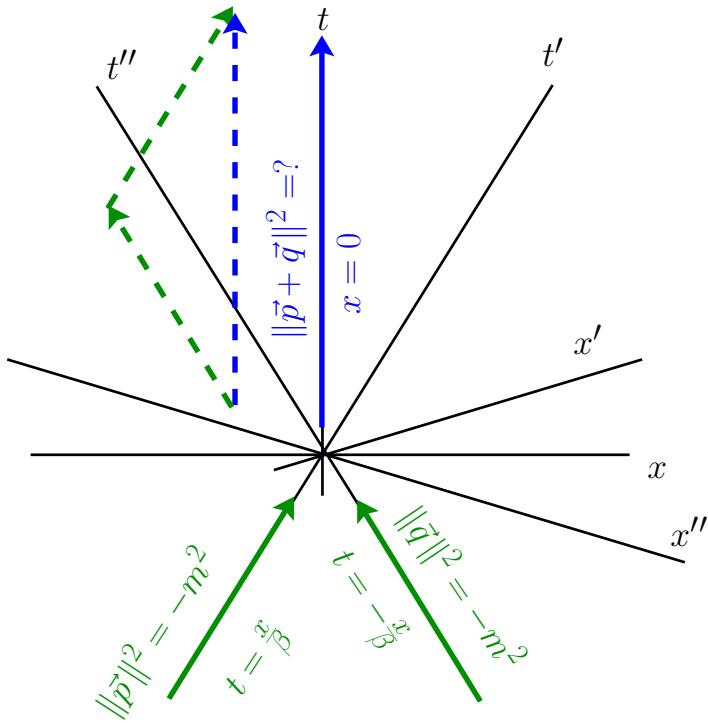
$$\vec{p} = m(\gamma, +\beta\gamma, 0, 0)^T$$

$$\vec{q} = m(\gamma, -\beta\gamma, 0, 0)^T$$

$$\vec{r} = \vec{p} + \vec{q}$$

$$L = m[2\gamma, (-\beta + \beta)\gamma, 0, 0]^T$$

SR: $\vec{p} \dots$: invariant or not?

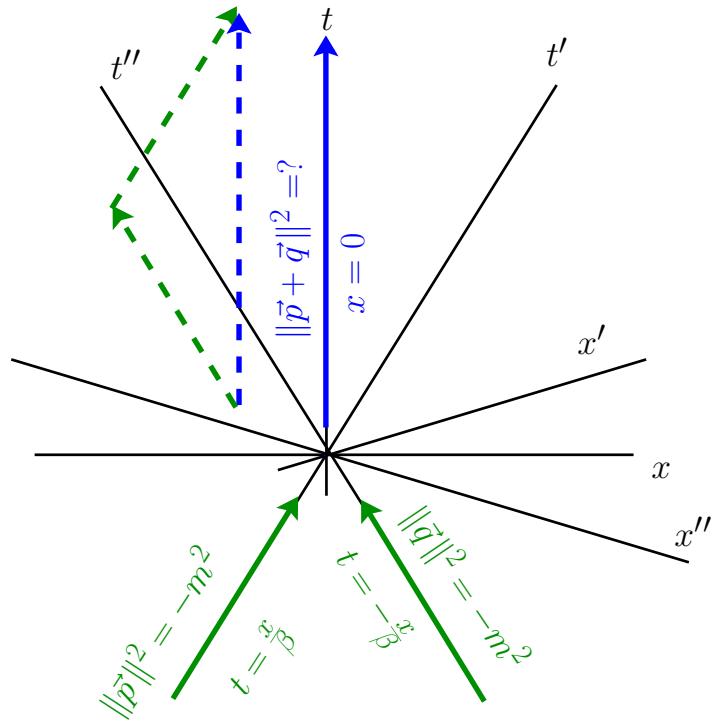


$$\vec{p} = m(\gamma, +\beta\gamma, 0, 0)^T$$

$$\vec{q} = m(\gamma, -\beta\gamma, 0, 0)^T$$

$$\vec{r} = \vec{p} + \vec{q} = 2m\gamma(1, 0, 0, 0)^T$$

SR: \vec{p} ... invariant or not?



system rest mass before and after: $2m\gamma$

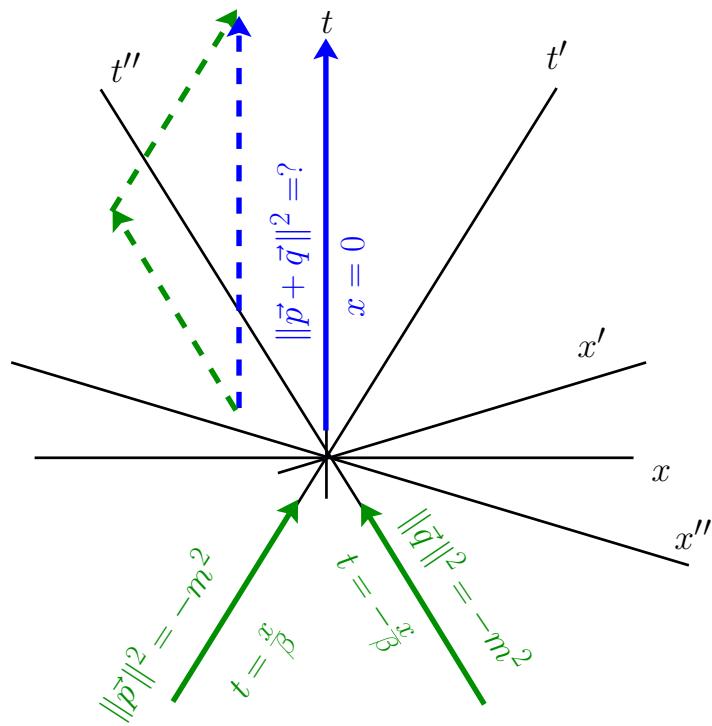
$$\vec{p} = m(\gamma, +\beta\gamma, 0, 0)^T$$

$$\vec{q} = m(\gamma, -\beta\gamma, 0, 0)^T$$

$$\vec{r} = \vec{p} + \vec{q} = 2m\gamma(1, 0, 0, 0)^T$$



SR: \vec{p} ...: invariant or not?



system rest mass before and after: $2m\gamma$
rest masses in many different frames:
 $m + m \neq 2m\gamma$ if $\gamma \neq 1$

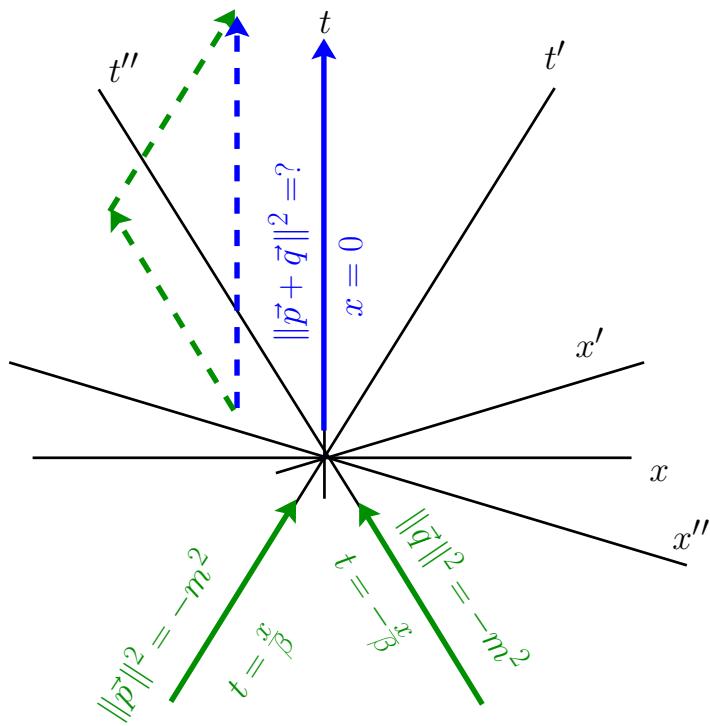
$$\vec{p} = m(\gamma, +\beta\gamma, 0, 0)^T$$

$$\vec{q} = m(\gamma, -\beta\gamma, 0, 0)^T$$

$$\vec{r} = \vec{p} + \vec{q} = 2m\gamma(1, 0, 0, 0)^T$$



SR: \vec{p} ...: invariant or not?



system rest mass before and after: $2m\gamma$
rest masses in many different frames:

$$m + m \neq 2m\gamma \text{ if } \gamma \neq 1$$

$$\sqrt{-\|\vec{p}\|^2} + \sqrt{-\|\vec{q}\|^2} \neq \sqrt{-\|\vec{r}\|^2}$$

$$\vec{p} = m(\gamma, +\beta\gamma, 0, 0)^T$$

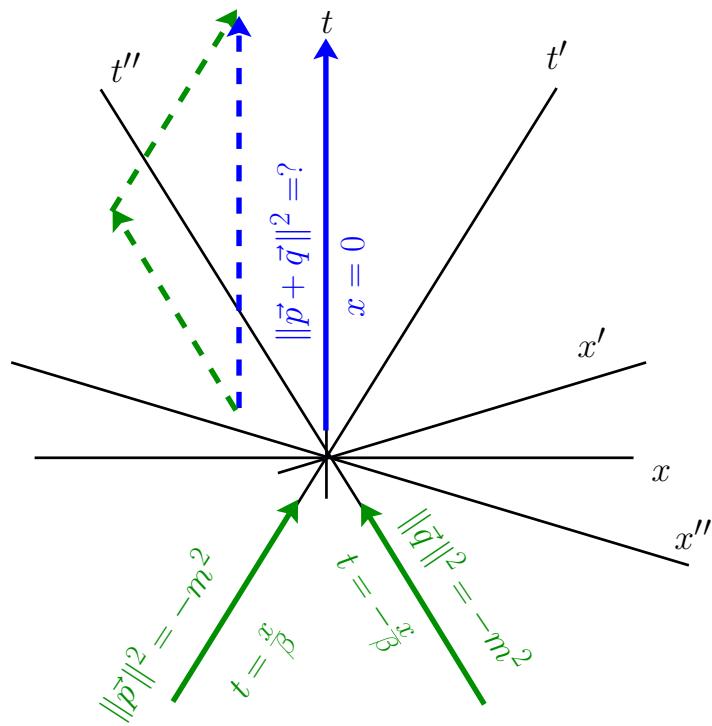
$$\vec{q} = m(\gamma, -\beta\gamma, 0, 0)^T$$

$$\vec{r} = \vec{p} + \vec{q} = 2m\gamma(1, 0, 0, 0)^T$$





SR: \vec{p} ...: invariant or not?



$$\vec{p} = m(\gamma, +\beta\gamma, 0, 0)^T$$

$$\vec{q} = m(\gamma, -\beta\gamma, 0, 0)^T$$

$$\vec{r} = \vec{p} + \vec{q} = 2m\gamma(1, 0, 0, 0)^T$$

system rest mass before and after: $2m\gamma$
rest masses in many different frames:

$$m + m \neq 2m\gamma \text{ if } \gamma \neq 1$$

$$\sqrt{-\|\vec{p}\|^2} + \sqrt{-\|\vec{q}\|^2} \neq \sqrt{-\|\vec{r}\|^2}$$

system mass is invariant, but can be divided into p^0 and $p^i, i \in \{1, 2, 3\}$ components in many different ways



SR: $\vec{p} \dots$: invariant or not?

interaction: momenta: $\vec{p} + \vec{q} \rightarrow \vec{r}$

moduli: $m, m, 2m\gamma$





SR: $\vec{p} \dots$: invariant or not?

interaction: momenta: $\vec{p} + \vec{q} \rightarrow \vec{r}$

moduli: $m, m, 2m\gamma$

total energy $E := p^0$





SR: $\vec{p} \dots$: invariant or not?

interaction: momenta: $\vec{p} + \vec{q} \rightarrow \vec{r}$

moduli: $m, m, 2m\gamma$

total energy $E := p^0$

rest frame: $E := p^0 = t$ component of $m(1, 0, 0, 0)^T$

$E = m$ (the famous equation)





SR: $\vec{p} \dots$: invariant or not?

interaction: momenta: $\vec{p} + \vec{q} \rightarrow \vec{r}$

moduli: $m, m, 2m\gamma$

total energy $E := p^0$

rest frame: $E := p^0 = t$ component of $m(1, 0, 0, 0)^T$

$E = m$ (the famous equation)

this means: in the rest frame, $K + m = m$ (trivial)





SR: $\vec{p} \dots$: invariant or not?

interaction: momenta: $\vec{p} + \vec{q} \rightarrow \vec{r}$

moduli: $m, m, 2m\gamma$

total energy $E := p^0$

rest frame: $E := p^0 = t$ component of $m(1, 0, 0, 0)^T$

$E = m$ (the famous equation)

this means: in the rest frame, $K + m = m$ (trivial)

more interesting: non-rest frame:





SR: $\vec{p} \dots$: invariant or not?

interaction: momenta: $\vec{p} + \vec{q} \rightarrow \vec{r}$

moduli: $m, m, 2m\gamma$

total energy $E := p^0$

rest frame: $E := p^0 = t$ component of $m(1, 0, 0, 0)^T$

$E = m$ (the famous equation)

this means: in the rest frame, $K + m = m$ (trivial)

more interesting: non-rest frame:

$$-m^2 = \|\vec{p}\|^2$$





SR: $\vec{p} \dots$: invariant or not?

interaction: momenta: $\vec{p} + \vec{q} \rightarrow \vec{r}$

moduli: $m, m, 2m\gamma$

total energy $E := p^0$

rest frame: $E := p^0 = t$ component of $m(1, 0, 0, 0)^T$

$E = m$ (the famous equation)

this means: in the rest frame, $K + m = m$ (trivial)

more interesting: non-rest frame:

$$-m^2 = \|\vec{p}\|^2$$

$$-m^2 = \|(E, p^x, p^y, p^z)^T\|^2$$





SR: $\vec{p} \dots$: invariant or not?

interaction: momenta: $\vec{p} + \vec{q} \rightarrow \vec{r}$

moduli: $m, m, 2m\gamma$

total energy $E := p^0$

rest frame: $E := p^0 = t$ component of $m(1, 0, 0, 0)^T$

$E = m$ (the famous equation)

this means: in the rest frame, $K + m = m$ (trivial)

more interesting: non-rest frame:

$$-m^2 = \|\vec{p}\|^2$$

$$-m^2 = \|(E, p^x, p^y, p^z)^T\|^2$$

$$-m^2 = -E^2 + p^2$$





SR: $\vec{p} \dots$: invariant or not?

interaction: momenta: $\vec{p} + \vec{q} \rightarrow \vec{r}$

moduli: $m, m, 2m\gamma$

total energy $E := p^0$

rest frame: $E := p^0 = t$ component of $m(1, 0, 0, 0)^T$

$E = m$ (the famous equation)

this means: in the rest frame, $K + m = m$ (trivial)

more interesting: non-rest frame:

$$-m^2 = \|\vec{p}\|^2$$

$$-m^2 = \|(E, p^x, p^y, p^z)^T\|^2$$

$$m^2 = E^2 - p^2$$





SR: $\vec{p} \dots$: invariant or not?

interaction: momenta: $\vec{p} + \vec{q} \rightarrow \vec{r}$

moduli: $m, m, 2m\gamma$

total energy $E := p^0$

rest frame: $E := p^0 = t$ component of $m(1, 0, 0, 0)^T$

$E = m$ (the famous equation)

this means: in the rest frame, $K + m = m$ (trivial)

more interesting: non-rest frame:

$$-m^2 = \|\vec{p}\|^2$$

$$-m^2 = \|(E, p^x, p^y, p^z)^T\|^2$$

$$m^2 = E^2 - \|(3)\vec{p}\|^2$$





SR: null 4-momentum

photon: $m = 0$





SR: null 4-momentum

photon: $m = 0$

extend defn of 4-momentum to \vec{p} for a photon





SR: null 4-momentum

photon: $m = 0$

extend defn of 4-momentum to \vec{p} for a photon

$\not\Rightarrow \vec{u} = \frac{\vec{p}}{m}$ since $m = 0$; no 4-velocity





SR: null 4-momentum

photon: $m = 0$

extend defn of 4-momentum to \vec{p} for a photon

$\not\Rightarrow \vec{u} = \frac{\vec{p}}{m}$ since $m = 0$; no 4-velocity

so $0 = E^2 - p^2$





SR: null 4-momentum

photon: $m = 0$

extend defn of 4-momentum to \vec{p} for a photon

$\nRightarrow \vec{u} = \frac{\vec{p}}{m}$ since $m = 0$; no 4-velocity

so $0 = E^2 - p^2$

i.e. $\vec{p} = (E, E, 0, 0)^T = (p, p, 0, 0)^T$ (if x direction)





SR: null 4-momentum

photon: $m = 0$

extend defn of 4-momentum to \vec{p} for a photon

$\not\Rightarrow \vec{u} = \frac{\vec{p}}{m}$ since $m = 0$; no 4-velocity

so $0 = E^2 - p^2$

i.e. $\vec{p} = (E, E, 0, 0)^T = (p, p, 0, 0)^T$ (if x direction)

so $E = p$ for any massless particle





SR: model summary

Minkowski spacetime: draw a correct diagram





SR: model summary

Minkowski spacetime: draw a correct diagram

Lorentz transformation (boost) $\Lambda(\phi)$ or $\Lambda(\beta)$





SR: model summary

Minkowski spacetime: draw a correct diagram

Lorentz transformation (boost) $\Lambda(\phi)$ or $\Lambda(\beta)$

refuse the assumption of absolute simultaneity (time)

