Szekeres models: a powerful theoretical tool in Cosmology.

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This talk is based on results obtained in the following articles:

1. arXiv:1701.00819 [pdf, other]

Non-Spherical Szekeres models in the language of Cosmological Perturbations

Roberto A. Sussman, Juan Carlos Hidalgo, Ismael Delgado Gaspar, Gabriel German

Comments: V2: Minor comments and a couple of references added. Version accepted for publication in PRD

Journal-ref: Phys. Rev. D 95, 064033 (2017)

Subjects: **General Relativity and Quantum Cosmology (gr-qc)**; Cosmology and Nongalactic Astrophysics (astro-ph.CO)

Phys. Rev. D **95**, 064033 (2017)

1. arXiv:1508.03127 [pdf, other]

Multiple non-spherical structures from the extrema of Szekeres scalars

Roberto A. Sussman, Ismael Delgado Gaspar

Comments: 27 pages, 9 figures. Typos corrected and references added

Subjects: **General Relativity and Quantum Cosmology (gr-qc)**; Cosmology and Nongalactic Astrophysics (astro-ph.CO);

Phys. Rev. D **92**, 083533 (2015)

2. arXiv:1507.02306 [pdf, other]

Coarse-grained description of cosmic structure from Szekeres models

Roberto A. Sussman, I. Delgado Gaspar, Juan Carlos Hidalgo

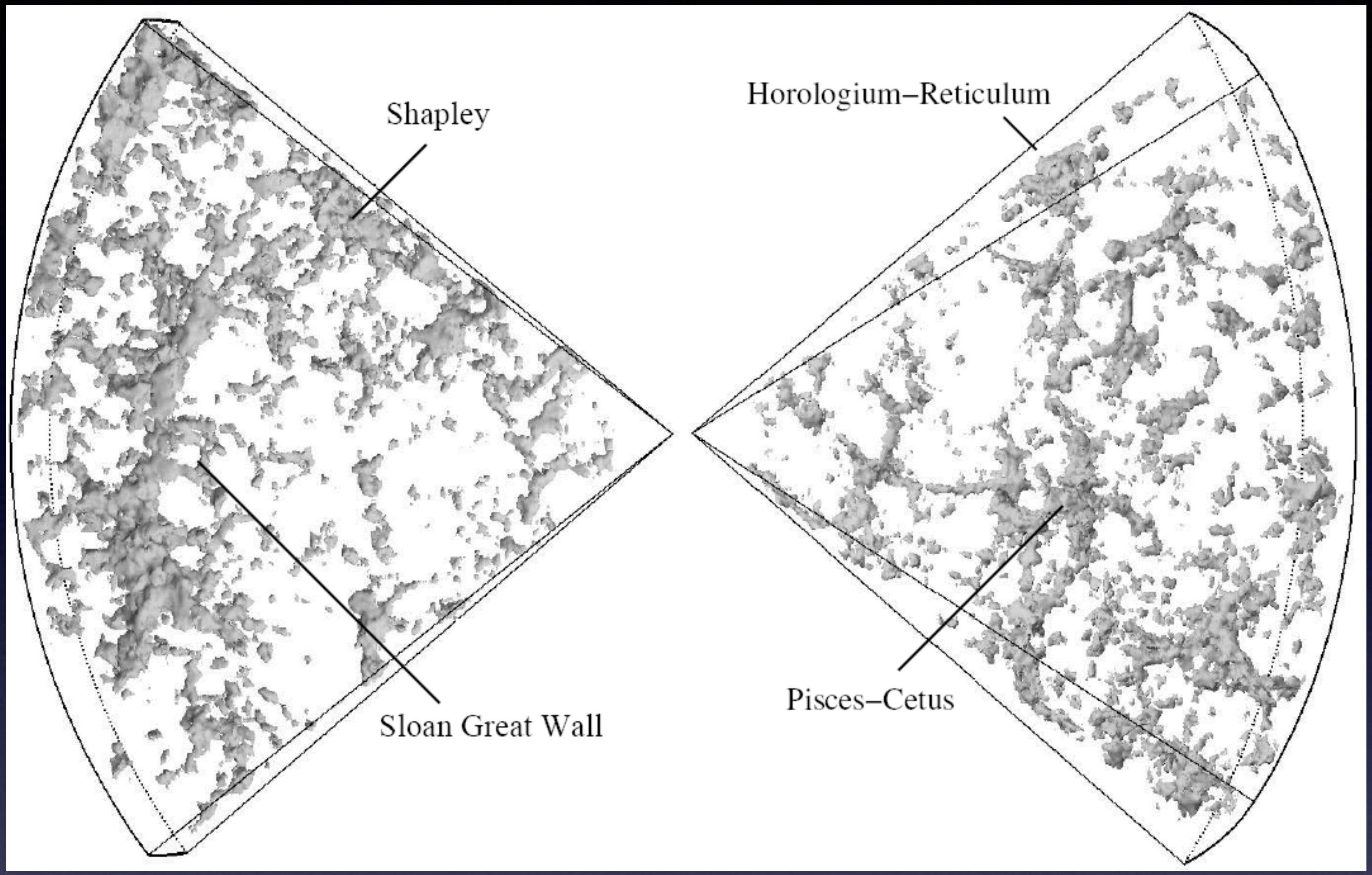
Comments: V3: Discussion of the expansion eigenvalues and of the Zeldovich approximation added. Figures modified accordingly. References updated. Version accepted for publication in JCAP

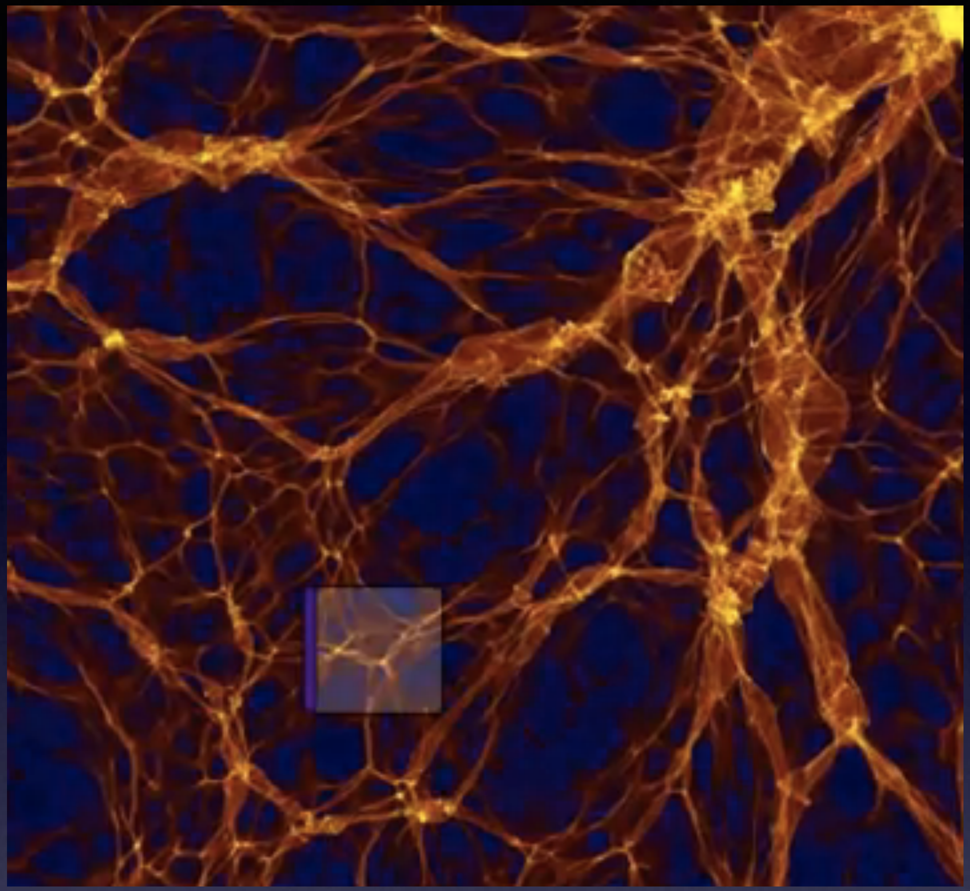
Subjects: **General Relativity and Quantum Cosmology (gr-qc)**; Cosmology and Nongalactic Astrophysics (astro-ph.CO)

JCAP

Quasi-spherical Szekeres models

Cosmic structure: web of voids, walls & filaments





Source: R. van de Weygaert & W Schaap, in Data Analysis and Cosmology (eds V Martínez, E Saar, E, Martínez-González, M. Pons-Bordería, Springer Verlag, Berlin, Lecture Notes on Physics 665 (2009) p 291

C.S. Frenk and S.D.M. White, Ann Phys, 524, 507534 (2012)

Galactic surveys involving hundreds of millions of galaxies

Newtonian n-body simulations provide a good description with lots of detail

Can we hope to provide a “decent” (at least coarse grained) description of cosmic structure with an exact solution of Einstein’s equations?

YES, with Szekeres models !!

It is easier to work in spherical coordinates

$$ds^2 - dt^2 + a^2 \left\{ \left[\frac{(\Gamma - W)^2}{1 - K_0 r^2} + W_1 \right] dr^2 + \frac{2W_2}{(1+\cos\theta)^2} dr d\theta + \frac{2W_3}{(1+\cos\theta)^2} dr d\phi + r^2 \left(d\theta^2 + \sin^2\theta d\phi^2 \right) \right\}$$

The Szekeres dipole W defines a precise direction in 3d

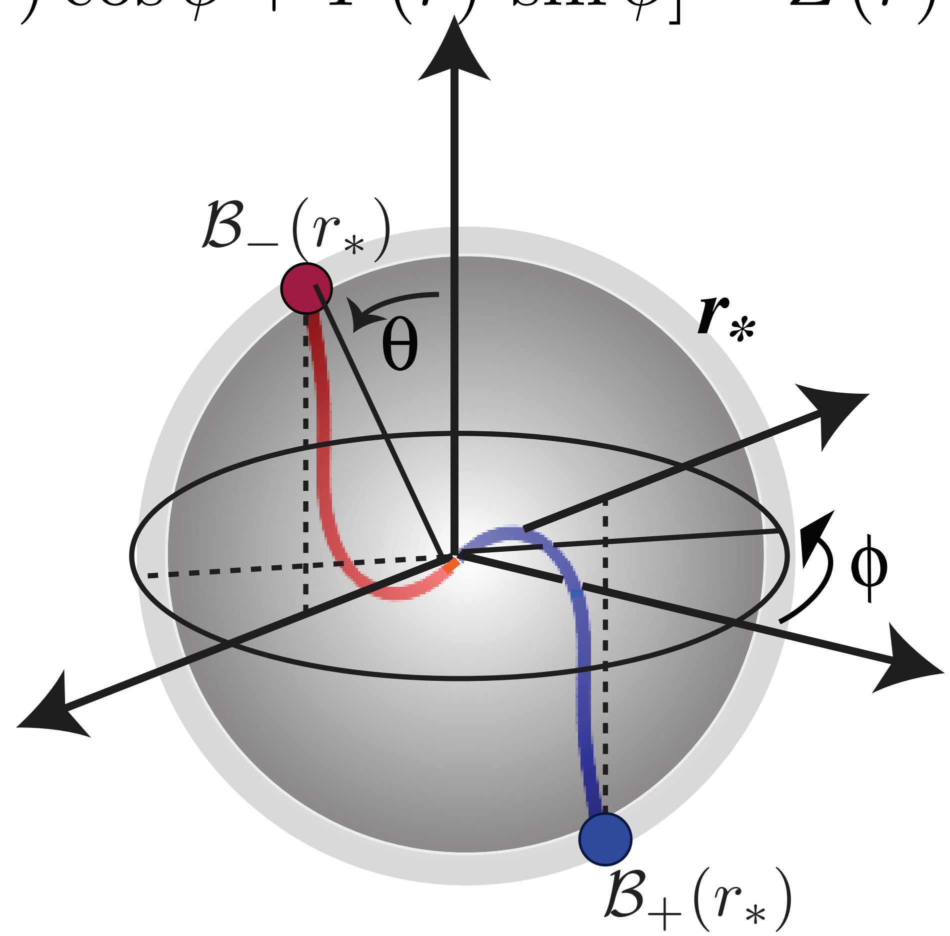
Angular extrema = extrema at each 2-sphere r = const.

$$W(r,\theta,\phi) = -\sin\theta \, [X(r)\cos\phi + Y(r)\, \sin\phi] - Z(r)\, \cos\theta,$$

$$\frac{\partial W}{\partial \theta} = \frac{\partial W}{\partial \phi} = 0$$

defines two curves that depend on the choice of X, Y, Z

$$\mathcal{B}_\pm(r) = [r, \theta_\pm(r), \phi_\pm(r)]$$



red curve = angular minima

Surfaces r constant are 2-spheres

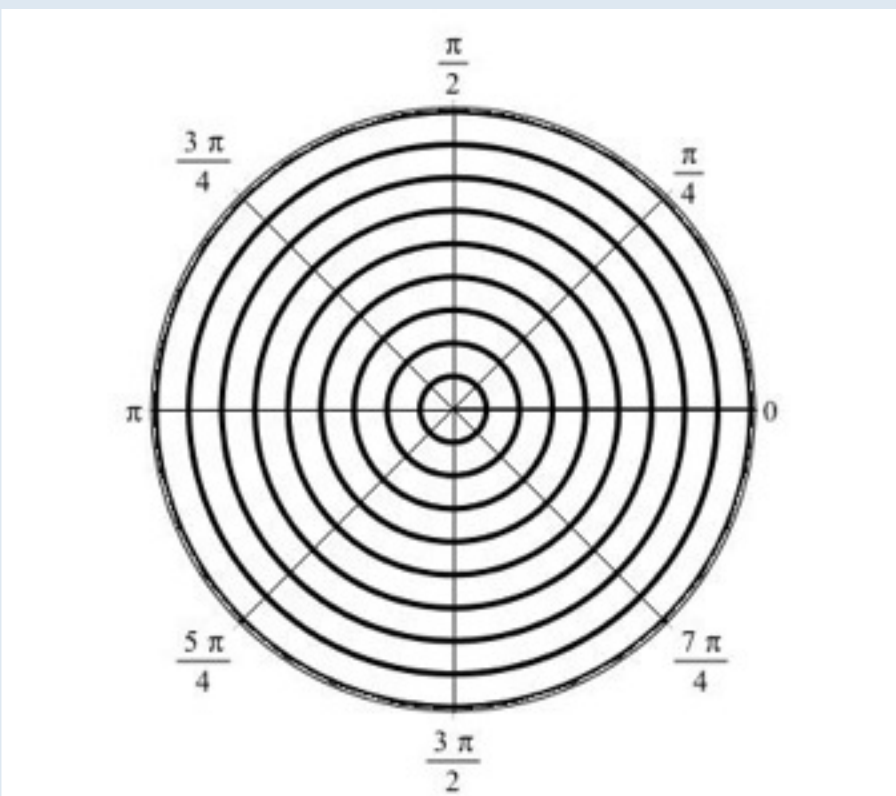
$$ds^2 - dt^2 + a^2 \left\{ \left[\frac{(\Gamma - W)^2}{1 - K_0 r^2} + W_1 \right] dr^2 + \frac{2W_2}{(1+\cos\theta)^2}\, dr\, d\theta + \frac{2W_3}{(1+\cos\theta)^2}\, dr\, d\phi + r^2 \left(d\theta^2 + \sin^2\theta\, d\phi^2 \right) \right\}$$

$$dt = dr = 0 \quad \Rightarrow$$

$$ds^2 = a^2\, r^2 \left(d\theta^2 + \sin^2\theta\, d\phi^2 \right)$$

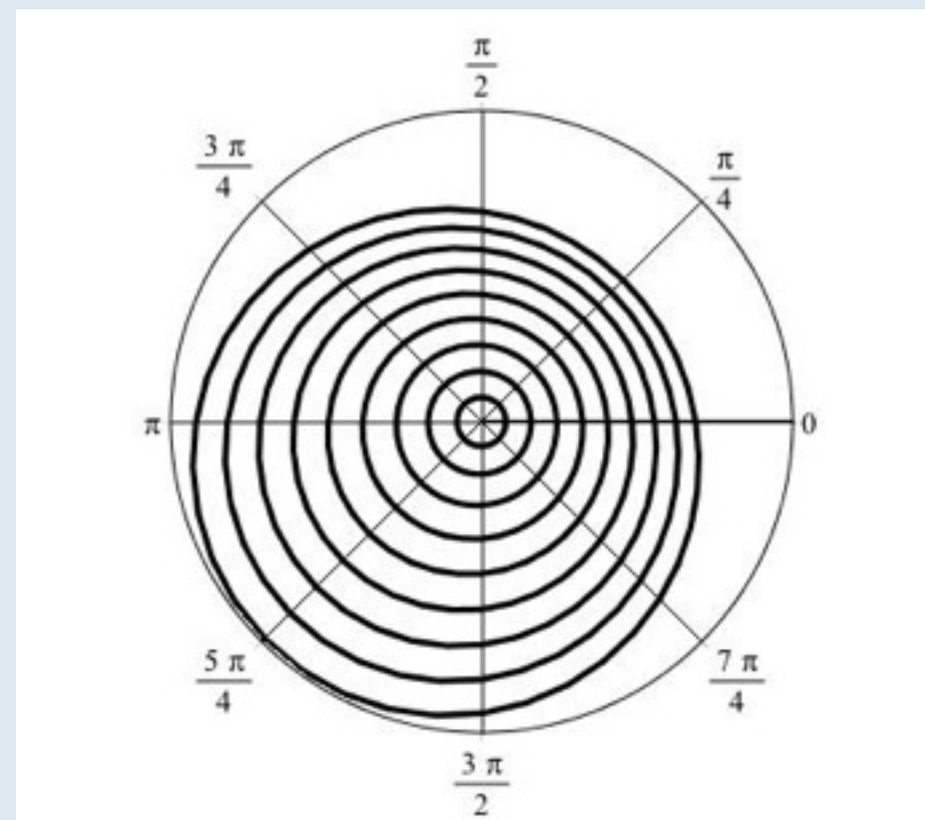
Non-sphericity — way in which 2-spheres foliate time slices

Compare proper radial length $\ell = \int_0^r \sqrt{g_{rr}}\, dr$



Spherical symmetry: CONCENTRIC 2-spheres

$\ell = \ell(r)$ radial rays are ORTHOGONAL to 2-spheres



Szekeres geometry: NON-CONCENTRIC 2-spheres

$\ell = \ell(r, \theta, \phi)$ radial rays are NOT ORTHOGONAL to 2-spheres

We now concentrate on the density:

Over-density = density maximum

Density void = density minimum

Transition = density saddle

Necessary & sufficient conditions for the 3-d spatial extrema of the density

$$\frac{\partial\,\rho}{\partial\theta} = \frac{\partial\,\rho}{\partial\phi} = \frac{\partial\,\rho}{\partial r} = 0 \qquad \text{at an arbitrary fixed } t$$

HOWEVER: Angular extrema of the density = Angular Extrema of the Dipole W

$$\left[\frac{\partial W}{\partial\theta}\right]_r = \left[\frac{\partial W}{\partial\phi}\right]_r = 0 \quad \Leftrightarrow \quad \left[\frac{\partial\rho}{\partial\theta}\right]_r = \left[\frac{\partial\rho}{\partial\phi}\right]_r = 0,$$

THEREFORE: Location of spatial extrema follows from the radial condition

$$\left[\frac{\partial\rho}{\partial r}\right]_{\pm} = 0$$

At r = 0 there can be a maximum or a minimum, depending on the sign of the second derivative. The classification of the extrema (maxima, minima, saddles) in r > 0 is more subtle (we look at this in the next slide)

IMPORTANT

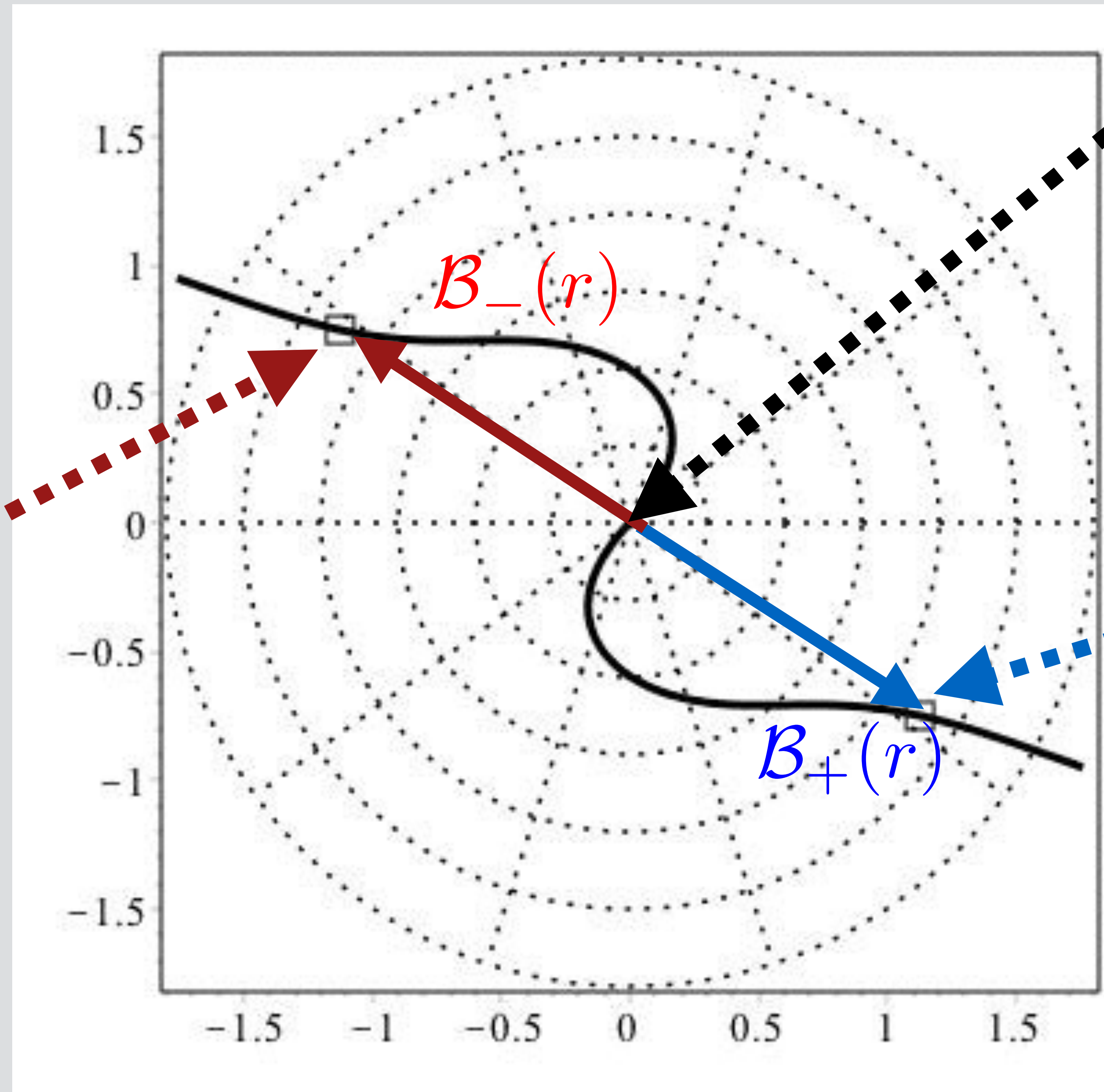
Radial conditions depend on time, therefore the 3-d spatial extrema of the density shift in time (they are not comoving)

Central density void (r = 0), at r > 0 an overdensity in $\mathcal{B}_+$ and a saddle in $\mathcal{B}_-$

Central density void (spatial density maximum)

$$\left[\frac{\partial \rho}{\partial r}\right]_{r=0} = 0,$$

$$\left[\frac{\partial^2 \rho}{\partial r^2}\right]_{r=0} > 0$$



Spatial saddle of ρ

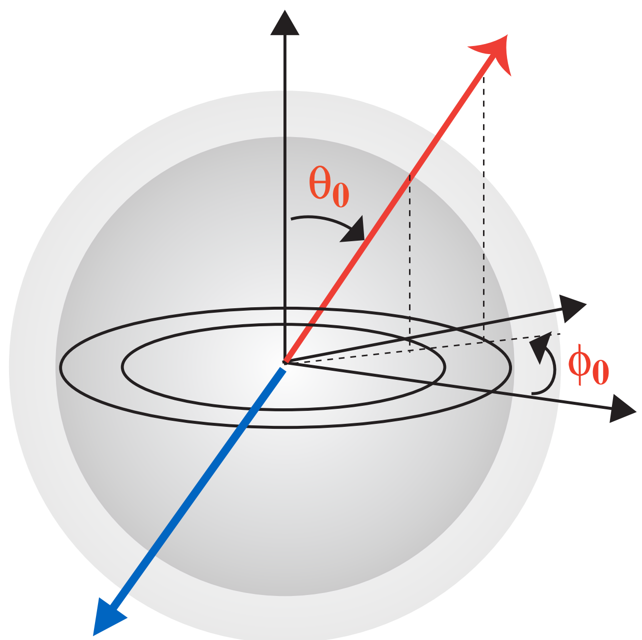
Radial maximum AND angular minimum

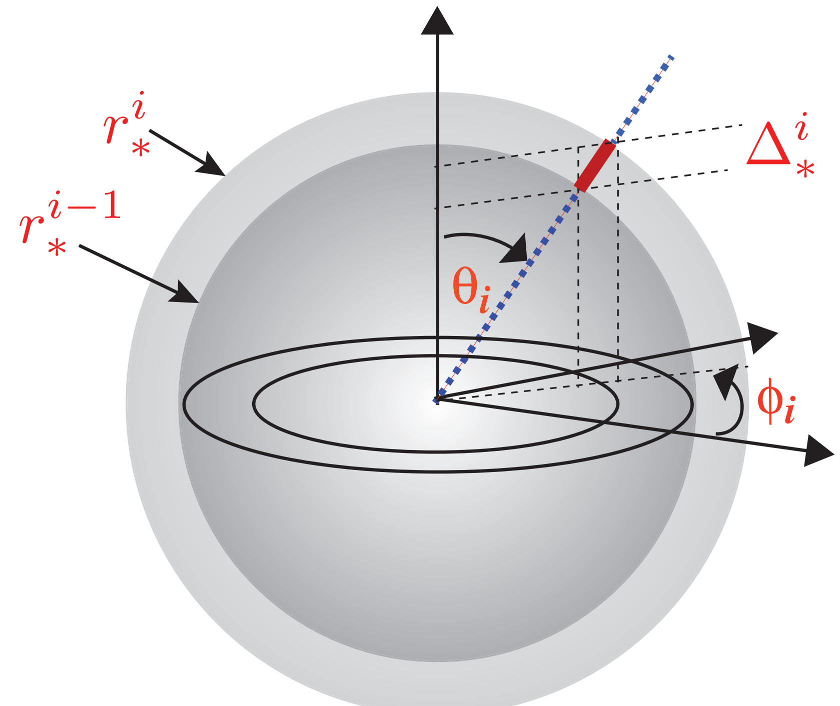
Density void (Spatial density minimum)

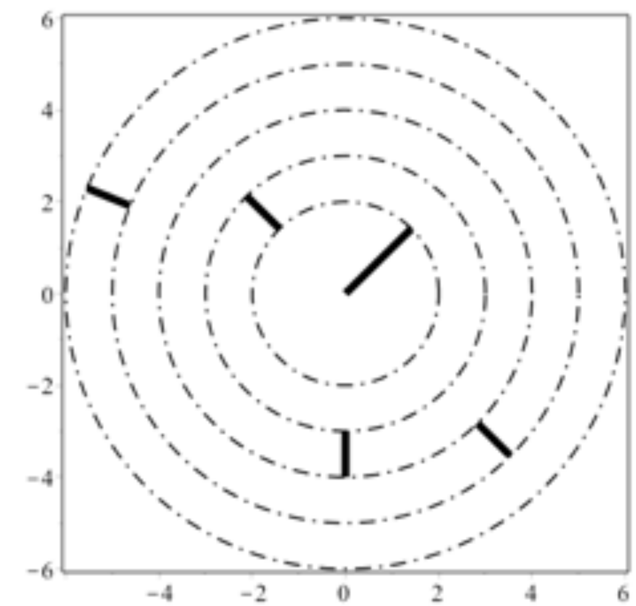
Radial maximum AND angular maximum

How can we obtain a precise location of the structures ?

If we define the dipole parameters as piecewise functions







Single direction for all r

$$X = a_0\, f(r)$$
$$Y = b_0\, f(r)$$
$$Z = c_0\, f(r)$$

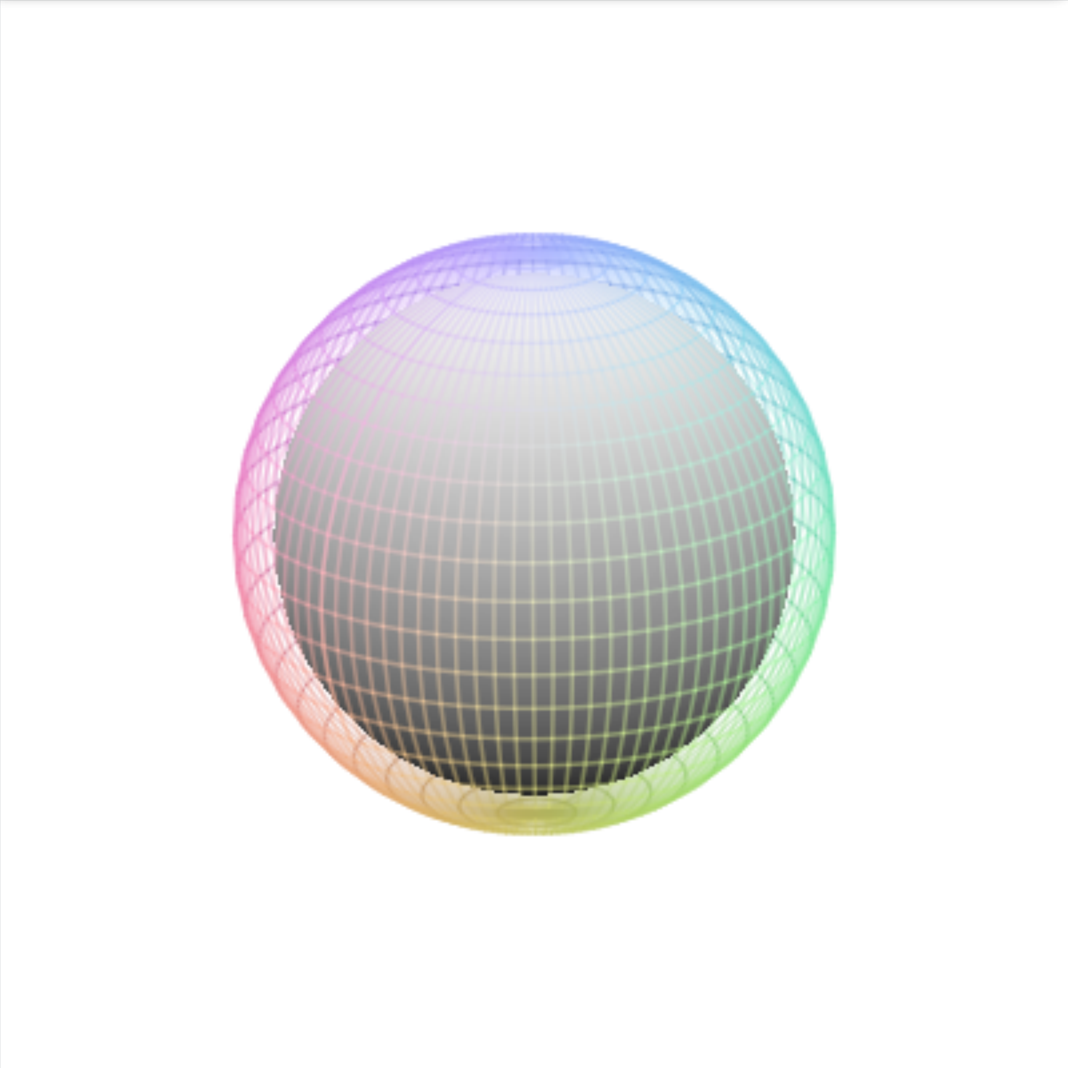
Different directions for each radial range

$$X = \begin{cases} a_{01}\, f_1, & \Delta_*^1 \\ a_{02}\, f_2, & \Delta_*^2, \\ \quad \cdots & \\ a_{0n}\, f_n, & \Delta_*^n, \end{cases} \quad Y = \begin{cases} b_{01}\, f_1, & \Delta_*^1 \\ b_{02}\, f_2, & \Delta_*^2, \\ \quad \cdots & \\ b_{0n}\, f_n, & \Delta_*^n, \end{cases} \quad Z = \begin{cases} c_{01}\, f_1, & \Delta_*^1 \\ c_{02}\, f_2, & \Delta_*^2, \\ \quad \cdots & \\ c_{0n}\, f_n, & \Delta_*^n, \end{cases}$$

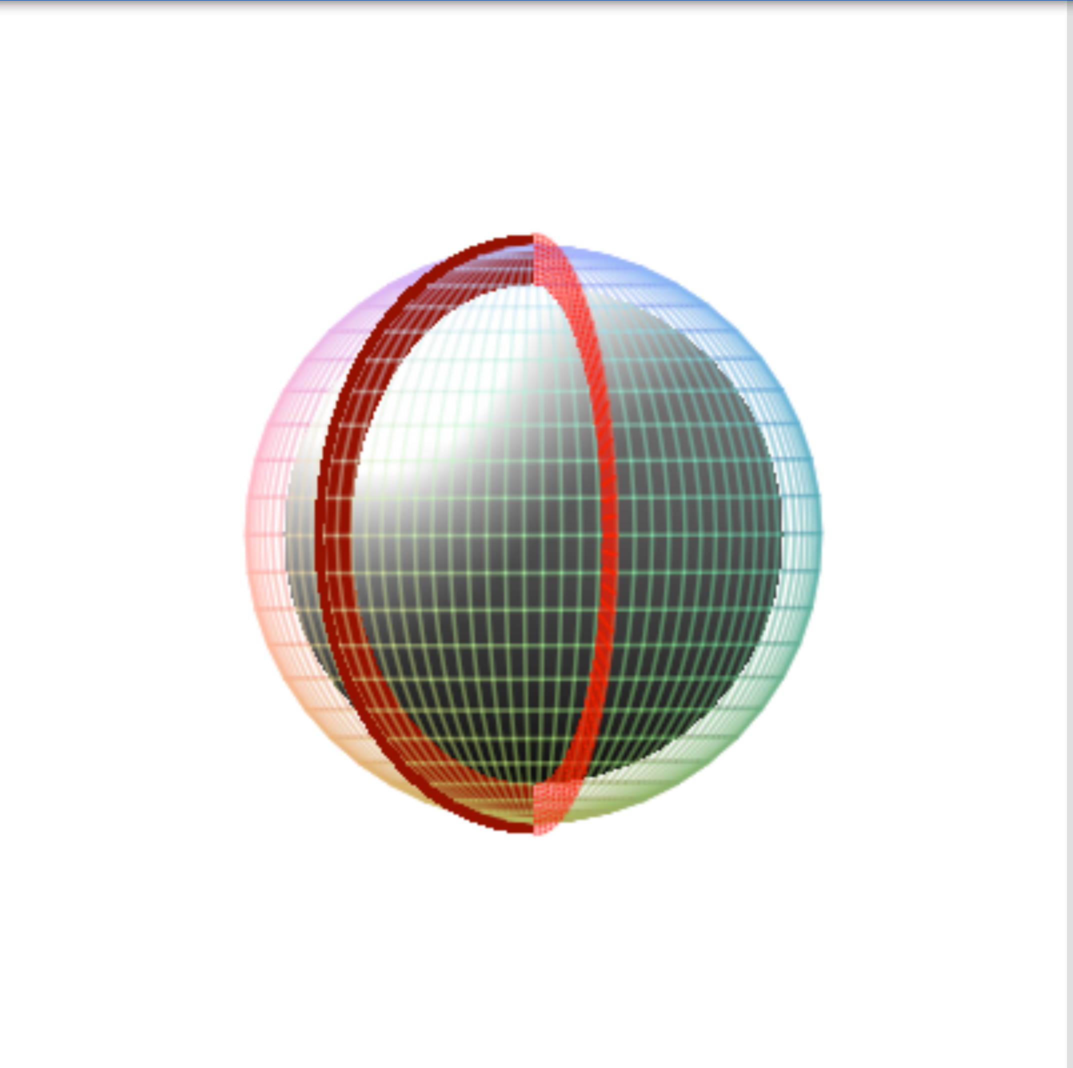
The over-densities & voids can be placed in arbitrary locations

Use piecewise functions to define the Dipole Parameters so that over-densities or voids are located in localised space partitions

Partitions in radial shells

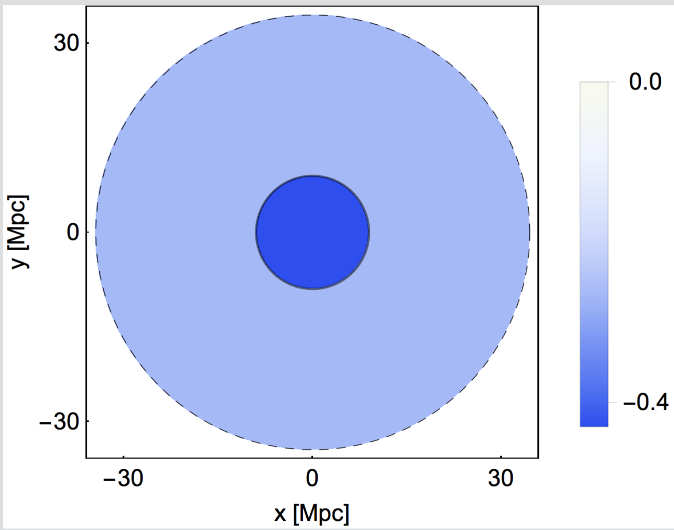


Angular partitions in each radial shell



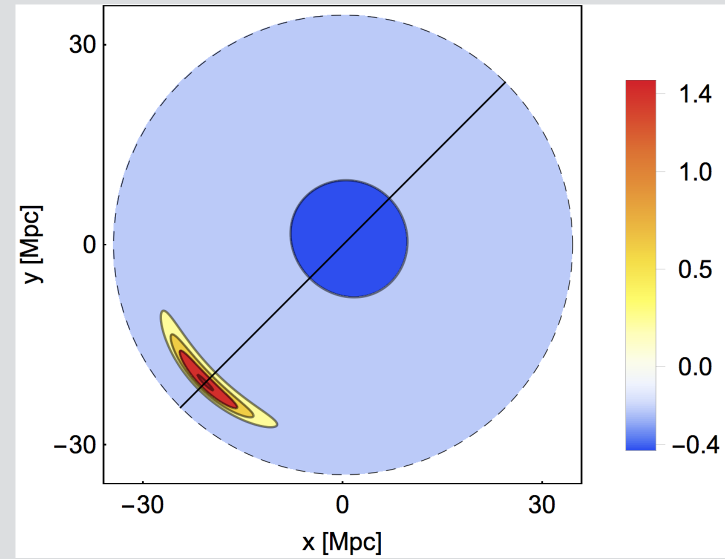
Junction conditions for angular partitions: only 1st form (metric) is continuous, THEREFORE must treat these partitions as a "thin shell approximation"

We know that Szekeres models can describe multiple structures



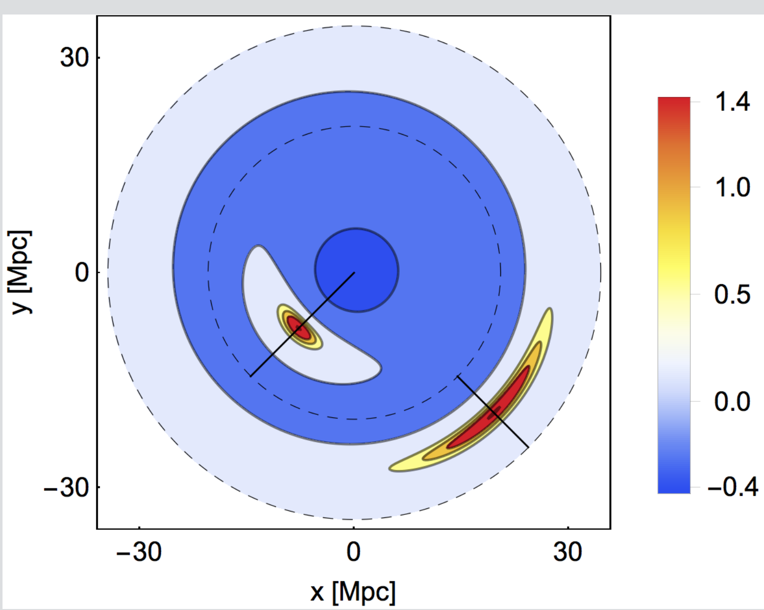
$$W = 0$$

Spherical Symmetry (zero dipole)
= one structure



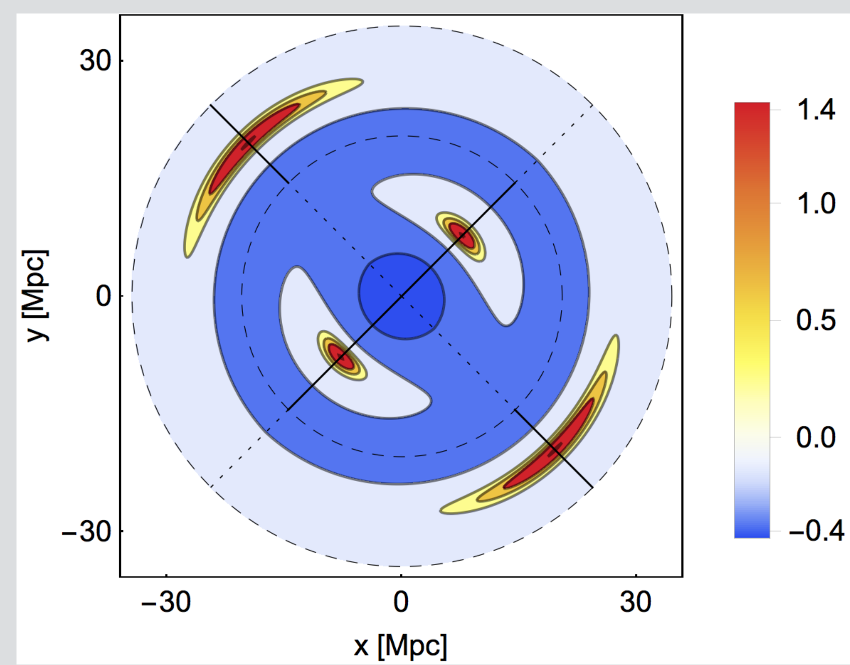
$$W = W(r, \theta, \phi) \; \forall \; (r, \theta, \phi)$$

Axial-like Symmetry
(simple dipole = 2 structures)



$$W = \text{radial piecewise in two intervals } \Delta_r$$

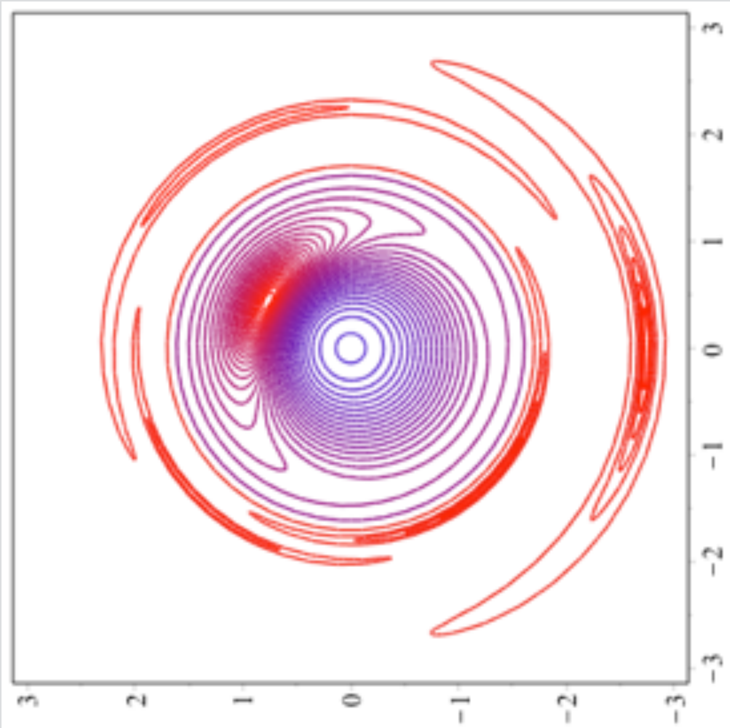
4 structures = 2 radially matched dipoles

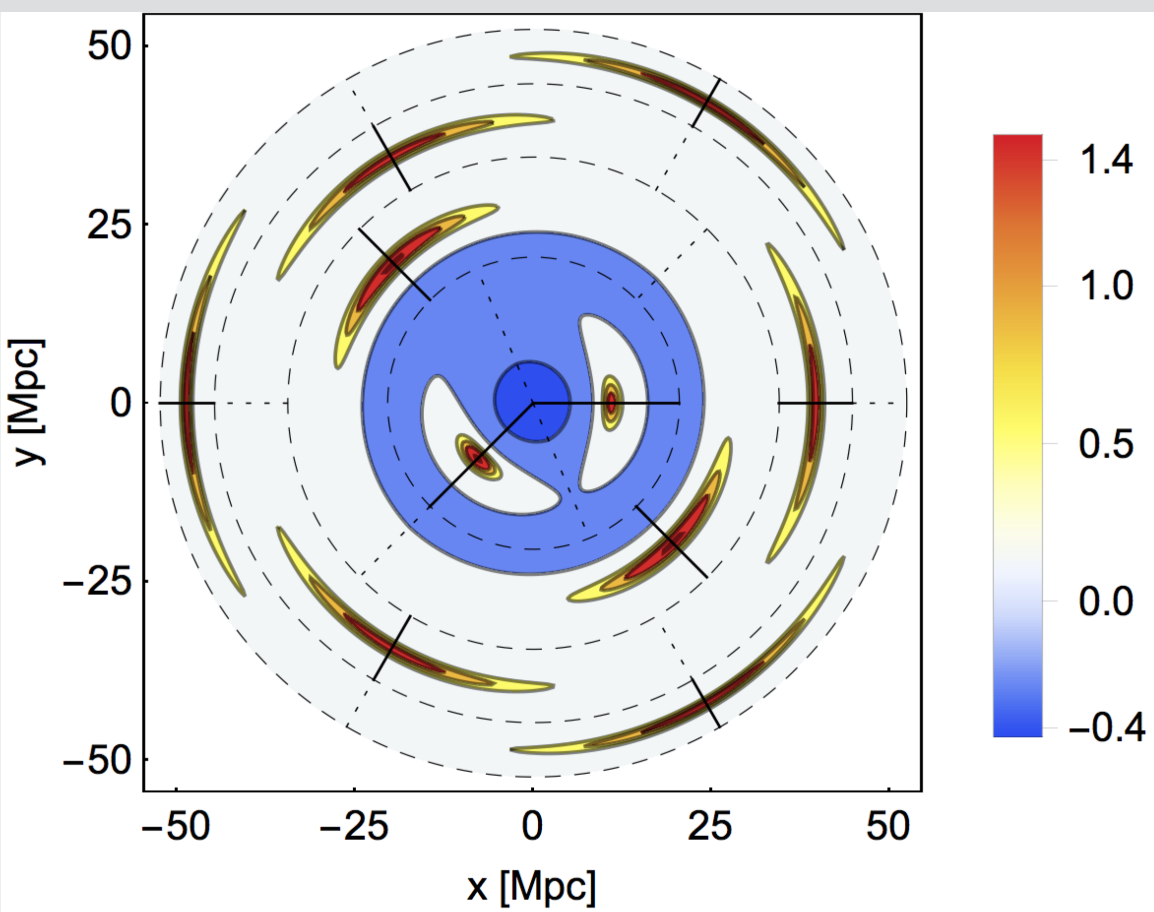


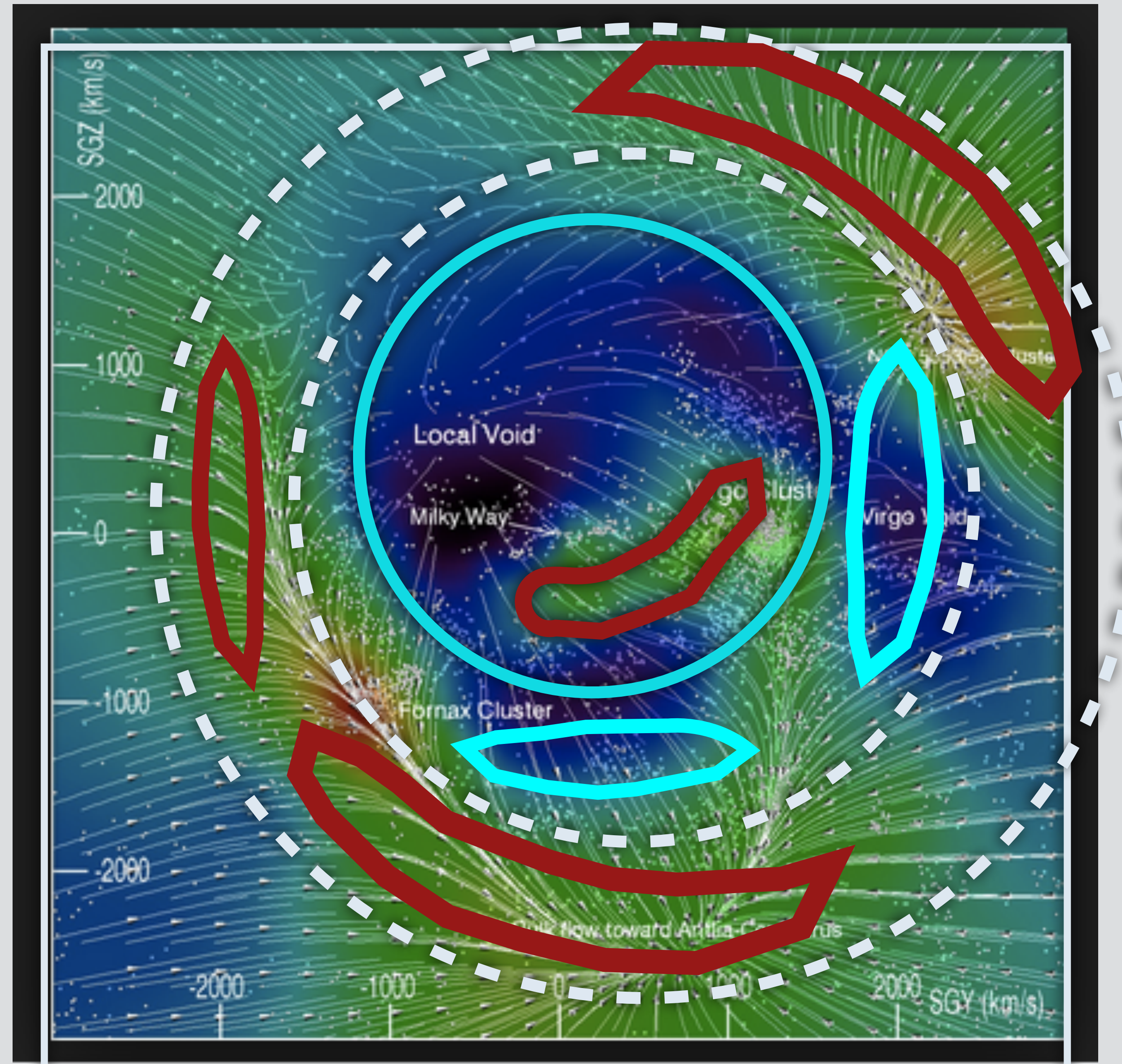
$$W = \text{radial \& angular piecewise in two intervals}$$

4 structures = 4 radial & angular matched dipoles

Coarse grained approximation (density contrast)







R. Brent Tully, Helene Courtois, Yehuda Hoffman and Daniel Pomarede, Nature 513, 71–73 (2014), "The Laniakea supercluster of galaxies".

Radial peculiar velocities with respect to the centre of the local void

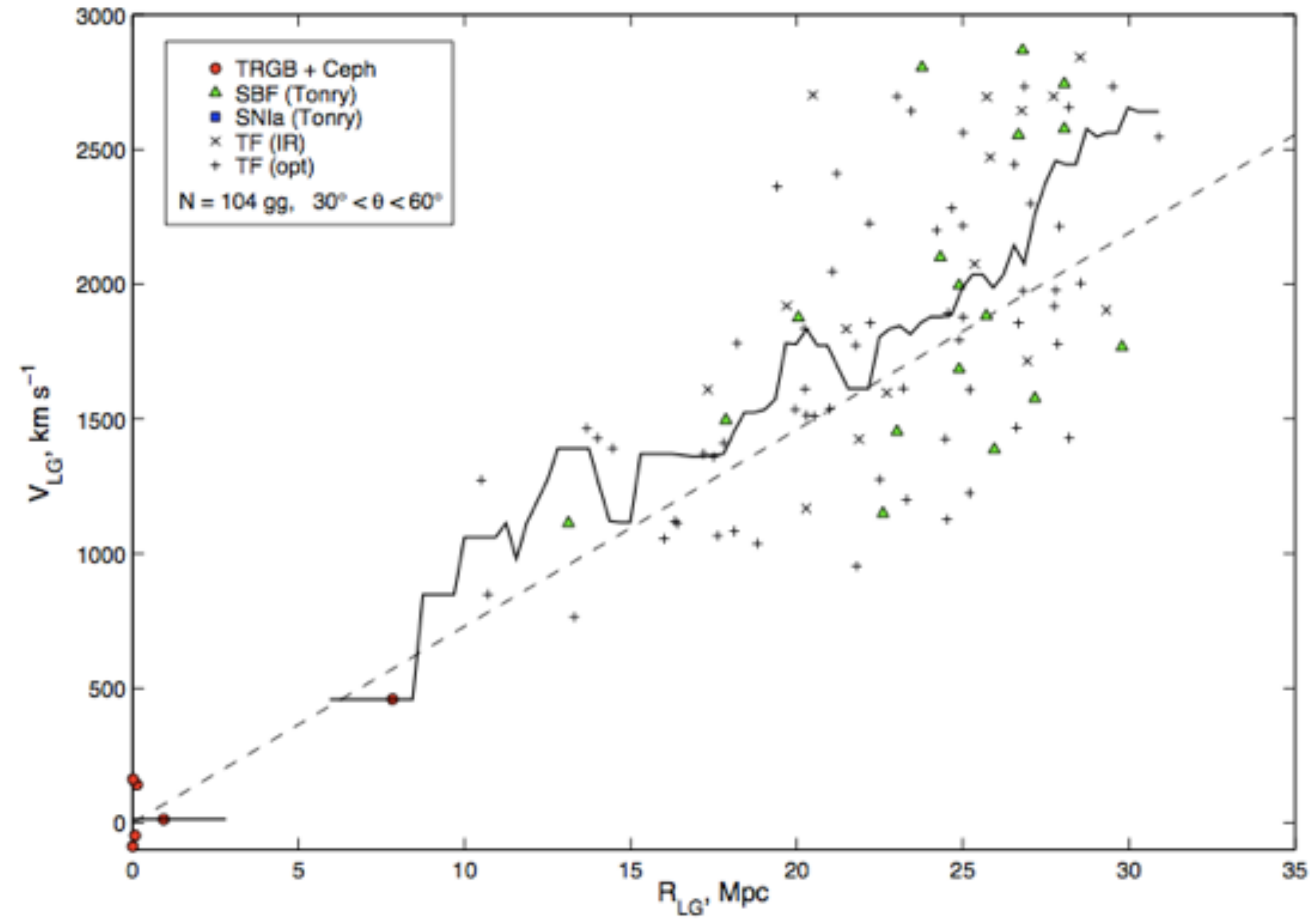
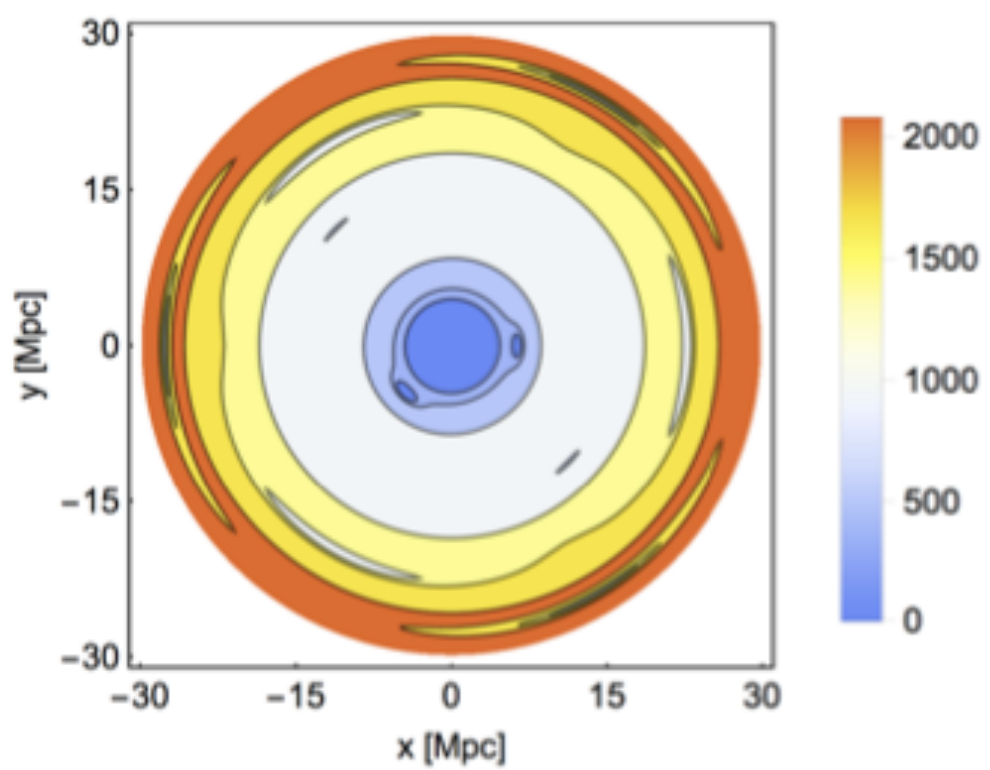


Fig. 4. Hubble diagram for galaxies with angular distances from the Local Void center below 30° (top panel) and 30 – 60° (bottom panel).



O.G. Nasonova and I.D. Karachentsev, Astrophysics, 54, 1-14 (2011) "On the kinematics of the Local cosmic void", arXiv:1011.5985v1 [astro-ph.CO]

Connection with Cosmological Perturbation Theory

Einstein's equations as dynamics of covariant objects with respect to a 4-velocity field

Covariant objects:

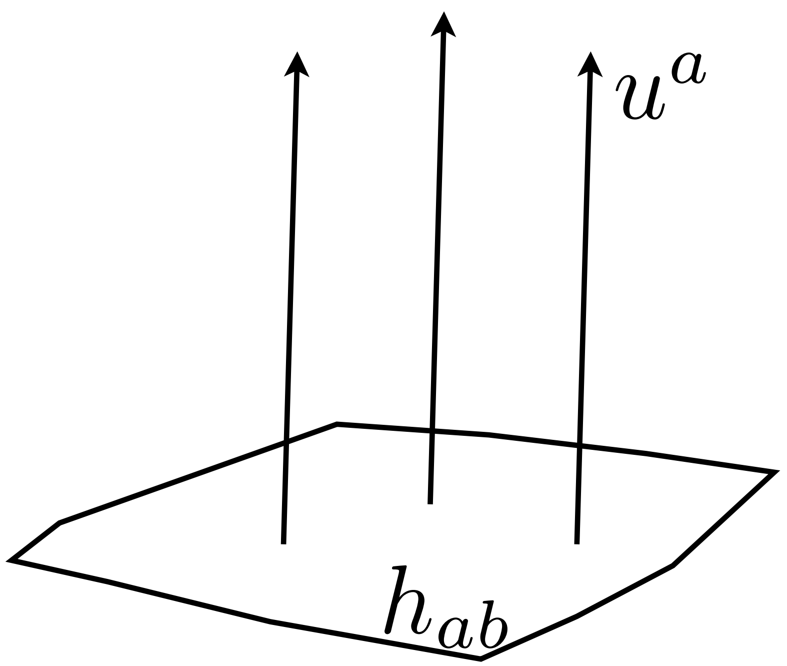
density $\rho,$

expansion $H \equiv \frac{\theta}{3}$

shear $\sigma_{ab} = \tilde{\nabla}_{(a} u_{b)} - H\, h_{ab}$

electric Weyl $E_{ab} = u^c u^d\, C_{acbd}$

spatial curvature $K \equiv \frac{^3\mathcal{R}}{6}$



Einstein's eqs as 1+3 Evolution eqs

$$\dot{\rho} = -3\, H\, \rho$$

$$\dot{H} = -H^2 - \frac{4\pi}{3}(\rho - 2\Lambda) + \frac{1}{3}\sigma_{ab}\sigma^{ab}$$

$$\dot{\sigma}_{\langle ab \rangle} = -2\, H\, \sigma_{ab} - \sigma_{\langle ac}\sigma^{c}_{b\rangle} - E_{ab}$$

$$\dot{E}_{\langle ab \rangle} = -3\, H\, E_{ab} - 4\pi\, \rho\, \sigma_{ab} + 3\, \sigma_{\langle ac}E^{c}_{b\rangle}$$

FLRW subset

$$\{\rho,\, H,\, K\}$$

Constraints

$$\tilde{\nabla}_b\, \sigma^b_a - 2\, \tilde{\nabla}_a H = 0$$

$$\tilde{\nabla}_b\, E^b_a - \tfrac{4\pi}{3}\, \tilde{\nabla}_a \rho = 0$$

$$H^2 = \frac{8\pi}{3}(\rho + \Lambda) - K + \sigma_{ab}\sigma^{ab}$$

Hamiltonian constraint

Change variables to averages & fluctuations of covariant scalars: Quasi-local scalars.

Covariant objects: $A = \rho,\ H,\ K$ **covariant scalars**

$\sigma_{ab},\ E_{ab}$ **traceless symmetric tensors**

- **Average function on spherical domains with non-trivial weight factor**

$$A_q = \frac{\int_{\mathcal{D}} A\, F\, dV_p}{\int_{\mathcal{D}} F\, dV_p} \qquad dV_p = \sqrt{\det\left(h_{ab}\right)}\, dr\, d\theta\, d\phi$$

→ **Will define the FLRW background**

- **Quasi local exact fluctuations** **relate "standard" and averaged scalars at the boundary of each spherical domain:**

$$\mathbf{D}^{(A)} = A - A_q, \qquad \Delta^{(A)} = \frac{A - A_q}{A_q}$$

→ **Will define the "perturbations" as exact objects**

Exact equations

$$\dot{\rho}_q = -3\rho_q H_q,$$
$$\dot{H}_q = -H_q^2 - \frac{4\pi}{3}\rho_q + \frac{8\pi}{3}\Lambda,$$
$$H_q^2 = \frac{8\pi}{3}(\rho_q + \Lambda) - K_q,$$

Background eqs

$$\dot{\Delta}^{(\rho)} = -3(1+\Delta^{(\rho)})\mathrm{D}^{(H)},$$
$$\mathrm{D}^{(H)} = (-2H_q + 3\mathrm{D}^{(H)})\mathrm{D}^{(H)} - \frac{4\pi}{3}\Delta^{(\rho)}$$

Perturbation equations

$$\mathbf{D}^{(K)} = \frac{8\pi}{3}\bar{\rho}\,\Delta^{(\rho)} - 2\bar{H}\,\mathbf{D}^{(H)}$$

Curvature perturbation

Linearised equations

$$\dot{\bar{\rho}} = -3\bar{\rho}\,\bar{H}$$
$$\dot{\bar{H}} = -\bar{H}^2 - \frac{4\pi}{3}\bar{\rho} + \frac{8\pi}{3}\Lambda$$
$$\bar{H}^2 = \frac{8\pi}{3}(\bar{\rho} + \Lambda) - \bar{K}$$

Background eqs

$$\dot{\Delta}^{(\rho)} = -3\mathbf{D}^{(H)}$$
$$\dot{\mathbf{D}}^{(H)} = -2\bar{H}\,\mathbf{D}^{(H)} - \frac{4\pi}{3}\bar{\rho}\,\Delta^{(\rho)}$$

Perturbation equations

$$\mathbf{D}^{(K)} = \frac{8\pi}{3}\bar{\rho}\,\Delta^{(\rho)} - 2\bar{H}\,\mathbf{D}^{(H)}$$

Curvature perturbation

Comparison with Linear Cosmological Perturbations

ΛCDM background:

$$ds^2 = -dt^2 + \bar{a}^2(t)\delta_{ij}dx^i dx^j, \qquad T^{ab} = \bar{\rho}u^a u^a + \Lambda g^{ab},$$

$$\dot{\bar{\rho}} = -3\bar{\rho}\bar{H}, \quad \dot{\bar{H}} = -\bar{H}^2 - \frac{4\pi}{3}(\bar{\rho} + 2\Lambda), \quad \bar{H}^2 = \frac{8\pi}{3}(\bar{\rho} + \Lambda) \qquad \bar{H} = \frac{\dot{\bar{a}}}{\bar{a}}, \quad \bar{\rho} = \frac{\bar{\rho}_i}{\bar{a}^3},$$

Perturbations (isochronous gauge):

$$\delta = \frac{\rho - \bar{\rho}}{\bar{\rho}}, \quad \vartheta = H - \bar{H}, \quad \mathcal{K} = K - \bar{K} \qquad \left\{|\delta|, \frac{|\vartheta|}{\bar{H}}, \mathcal{K}\bar{H}^2\right\} \ll 1,$$

$$ds^2 = -dt^2 + \bar{a}^2(t)\left[(1-2\psi)\delta_{ij} + \chi_{ij}\right]dx^i dx^j, \quad \chi_{ij} = \left(\partial_i\partial_j - \frac{1}{3}\delta_{ij}\nabla^2\right)\chi,$$

$$\vartheta = -\frac{3}{\bar{a}}\dot{\psi}, \qquad \mathcal{K} = 4\nabla^2\left(\psi + \frac{1}{6}\nabla^2\chi\right), \qquad \frac{3}{2}\mathcal{K} = 4\pi\delta - \bar{H}\vartheta,$$

Evolution equations:

$$\dot{\delta} = -\vartheta,$$
$$\dot{\vartheta} = -2\bar{H}\vartheta - 4\pi\bar{\rho}\delta,$$
$$\Rightarrow \quad \ddot{\delta} + 2\bar{H}\dot{\delta} - 4\pi\bar{\rho}\delta = 0,$$

identify:

$$\delta \leftrightarrow \Delta^{(\rho)},$$
$$\vartheta \leftrightarrow 3\mathrm{D}^{(H)},$$

Identical to:

$$\dot{\Delta}^{(\rho)} = -3\mathbf{D}^{(H)}$$
$$\dot{\mathbf{D}}^{(H)} = -2\bar{H}\,\mathbf{D}^{(H)} - \frac{4\pi}{3}\bar{\rho}\,\Delta^{(\rho)}$$
$$\ddot{\Delta}^{(\rho)} + 2H_q\dot{\Delta}^{(\rho)} - 4\pi\rho_q\Delta^{(\rho)} = 0$$

Reference: Bruni et al, Astrophys.J. 785 (2014) 2, arXiv 1307.1478v2

Comparison of solutions

Assume linear conditions at last scattering time (near homogeneity and near spatial flatness)

$$\Omega^m_{\rm ls} \approx 1, \quad |\Omega^k_{\rm ls}| \ll 1, \quad \Omega^\Lambda_{\rm ls} = \left(\frac{\bar{H}_0}{\bar{H}_{\rm ls}}\right)^2 \Omega^\Lambda_0 \sim 10^{-9}, \quad \bar{a} \sim t^{2/3}$$

Linearised Szekeres perturbation with suppressed decaying mode

$$\Delta^{(\rho)} = C_+(\vec{x})\, t^{2/3}, \qquad C_+ = C_+(r, \theta, \phi)$$

Linear cosmological density perturbation (decaying mode suppressed)

$$\delta = C(\vec{x})\, t^{2/3}, \qquad C(\vec{x}) = \frac{1}{L}\int_{-\infty}^{\infty} C_k(\vec{k}) \mathrm{e}^{-i\vec{k}\cdot\vec{x}} d^3k$$

Deterministic initial condition vs superposition of random modes

Obviously δ is much more general than $\Delta^{(\rho)}$, which must comply with constraints of Szekeres models. However, we know how $\Delta^{(\rho)}$ evolves well into the non-linear regime, whereas δ is only valid in the linear regime

Connection with Newtonian Gravity

The Szekeres collapse: an exact relativistic analogue of the Zeldovich approximation

Zeldovich proposed a first order correction in the retain between Eulerian x^i and Lagrangian y^i coordinates

$$y^i = \bar{a}(t)\left[x^i + \Psi^i(t, x^j)\right] \quad \textbf{where} \quad \Psi^i(t_0, x^j) = 0$$

Assuming that derivatives $\Psi^i_{\ ,j}$ are symmetric, the density takes the from

$$\rho = \frac{\rho_0}{\det(y^i_{\ ,j})} = \frac{\rho_0}{\bar{a}^3\left[1-\xi^{(1)}\right]\left[1-\xi^{(2)}\right]\left[1-\xi^{(3)}\right]}$$

where $-\xi^{(A)}\ (A = 1, 2, 3)$ are the eigenvalues of the "deformation" tensor $\xi^i_j = \Psi^i_{\ ,j}$

Considering that the $\xi^{(A)}$ are in general unequal, we can assume $0 \leq \xi^{(3)} \leq \xi^{(2)} \leq \xi^{(1)}$, leading to the following collapse ($\det(y^i_{\ ,j}) \to 0$) morphologies

Pancake collapse: $\xi^{(1)} \to 1 \quad \text{while} \quad \xi^{(2)} < \xi^{(3)} < 1,$

One direction collapses, two directions expand

Filamentary collapse: $\xi^{(1)}, \xi^{(2)} \to 1 \quad \text{while} \quad \xi^{(3)} < 1,$

Two direction collapse, one directions expands

Spherical or isotropic collapse: $\xi^{(1)}, \xi^{(2)}, \xi^{(3)} \to 1,$

Three directions collapse

What happens in Szekeres models?

Looking at Szekeres metric we identify to "scale factors" $a(t,r),\ \Gamma(t,r)$

$$ds^2 = -dt^2 + a^2 \left\{ \left[\frac{(\Gamma - W)^2}{1 - K_0 r^2} + W_1 \right] dr^2 + \frac{2W_2}{1+\cos^2\theta}\, dr\, d\theta + \frac{2W_3}{1+\cos^2\theta}\, dr\, d\phi + r^2 \left(d\theta^2 + \sin^2\theta d\phi^2\right) \right\}$$

Szekeres collapse/expansion is (in general) anisotropic. It is governed by the expansion tensor

$$H^a_b = \nabla_{(a} u_{b)} = H\, h^a_b + \sigma^a_b$$

whose eigenvalues are

$$\mathbf{H}^{(1)} = H_q + 3\mathbf{D}^{(H)} = \frac{\dot a}{a} + \frac{\dot\Gamma}{\Gamma - W} \qquad \mathbf{H}^{(2)} = \mathbf{H}^{(3)} = H_q = \frac{\dot a}{a}$$

scale factors $\ell^{(A)}$ **such that** $\dot\ell^{(A)}/\ell^{(A)} = \mathbf{H}^{(A)}$ **and** $\ell_0^{(A)} = 1$

$$\ell^{(1)} = \frac{a(\Gamma - W)}{1 - W}, \qquad \ell^{(2)} = \ell^{(3)} = a,$$

the Szekeres density takes the form

$$\rho = \frac{\rho_0\, \mathcal{J}_0}{\mathcal{J}} = \frac{\rho_0}{\ell^{(1)}\,\ell^{(2)}\,\ell^{(3)}}, \qquad \mathcal{J} = \sqrt{\det(g_{ij})}$$

Comparison between Szekeres & Zeldovich densities

$$\rho = \frac{\rho_0}{\ell^{(1)}\,\ell^{(2)}\,\ell^{(3)}} \quad \text{vs} \quad \rho = \frac{\rho_0}{\bar{a}^3\left[1-\xi^{(1)}\right]\left[1-\xi^{(2)}\right]\left[1-\xi^{(3)}\right]}$$

leads to the interpretation of the $\xi^{(A)}$ as the rate between the scale factors of Szekeres and the scale factor of the background $\bar{a}(t)$

$$\xi^{(1)} = 1 - \frac{\ell^{(1)}}{\bar{a}} = 1 - \frac{a}{\bar{a}} \times \frac{\Gamma - W}{1 - W}, \qquad \xi^{(2)} = \xi^{(3)} = 1 - \frac{\ell^{(2)}}{\bar{a}} = 1 - \frac{a}{\bar{a}}$$

The "deformation" tensor is expressible in terms of the orthonormal triad of the diagonal metric

$$\xi^i_j = \xi^{(||)}\mathbf{e}^i_{(1)}\mathbf{e}^{(1)}_j + \xi^{(\perp)}\left[\mathbf{e}^i_{(2)}\mathbf{e}^{(2)}_j + \mathbf{e}^i_{(3)}\mathbf{e}^{(3)}_j\right], \qquad \xi^{(||)} = \xi^{(1)}, \quad \xi^{(\perp)} = \xi^{(2)} = \xi^{(3)},$$

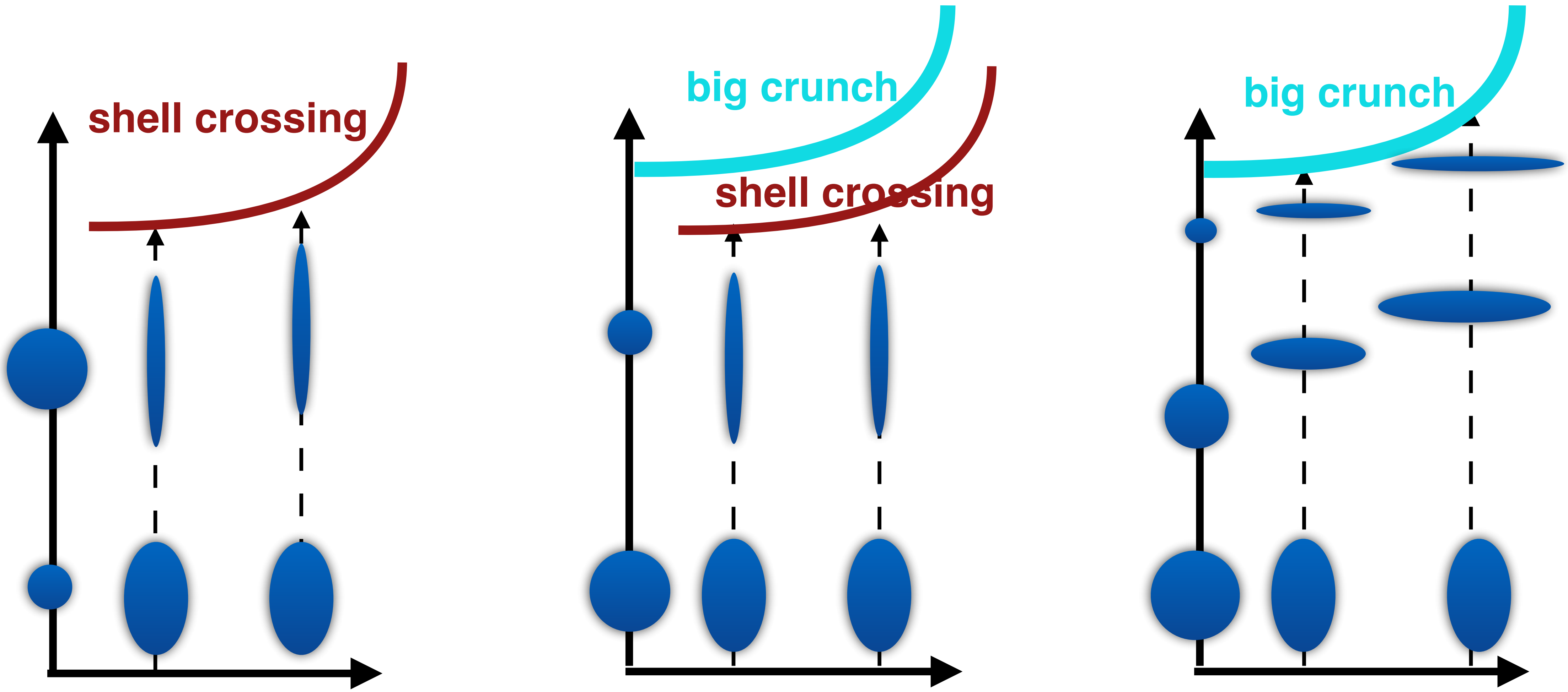
We have the following collapse morphologies

Pancake collapse: $\xi^{(||)} \to 1, \quad \xi^{(\perp)} < 1$ — **Collapse in the direction of triad vector $\mathbf{e}^i_{(1)}$ expansion in $\mathbf{e}^i_{(2)},\ \mathbf{e}^i_{(3)}$**

Filamentary collapse: $\xi^{(\perp)} \to 1, \quad \xi^{(||)} < 1$ — **Collapse in the directions $\mathbf{e}^i_{(2)},\ \mathbf{e}^i_{(3)}$ expansion in $\mathbf{e}^i_{(1)}$**

Spherical collapse: $\xi^{(\perp)},\ \xi^{(||)} \to 1,$ — **Only occurs at r = 0 for which** $\Gamma = 1,\ W = 0 \quad \Rightarrow \quad \xi^{(\perp)} = \xi^{(||)},$

How do these structures expand or collapse



Pancake collapse (shell crossings) in an expanding background

Isotropic expansion at r = 0

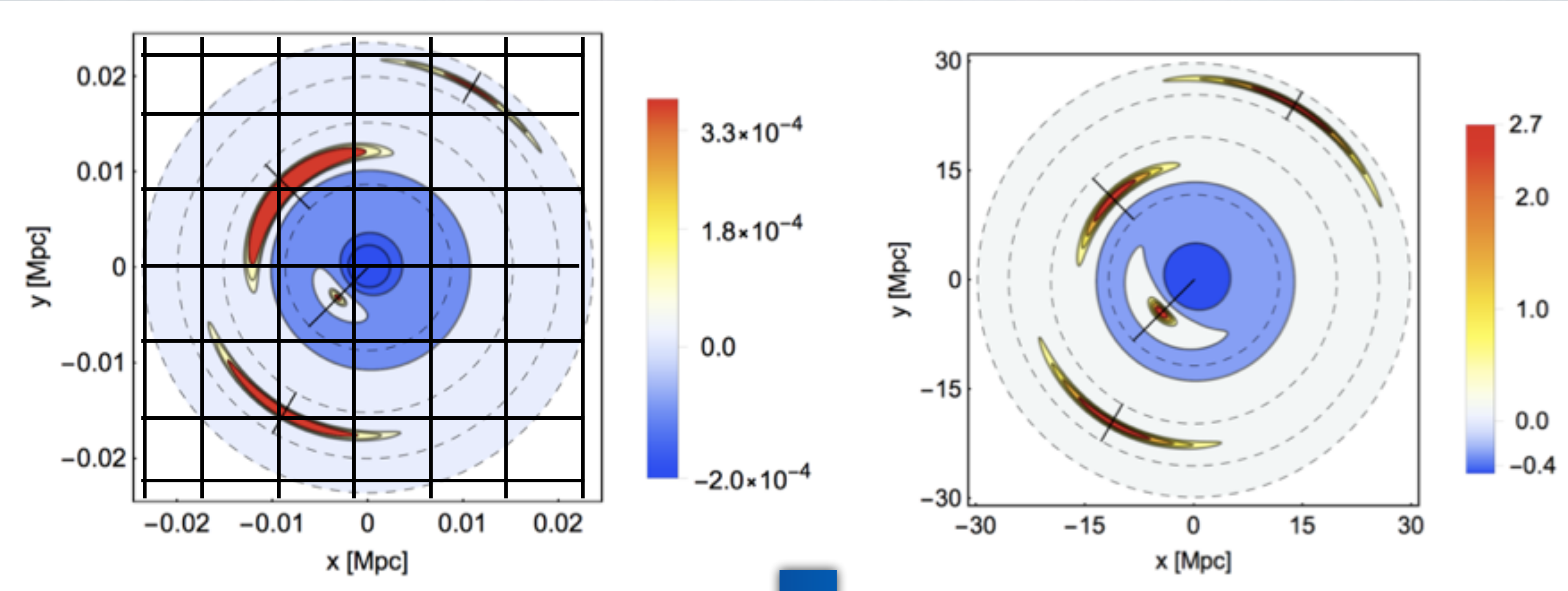
Pancake collapse (shell crossings) in a collapsing background

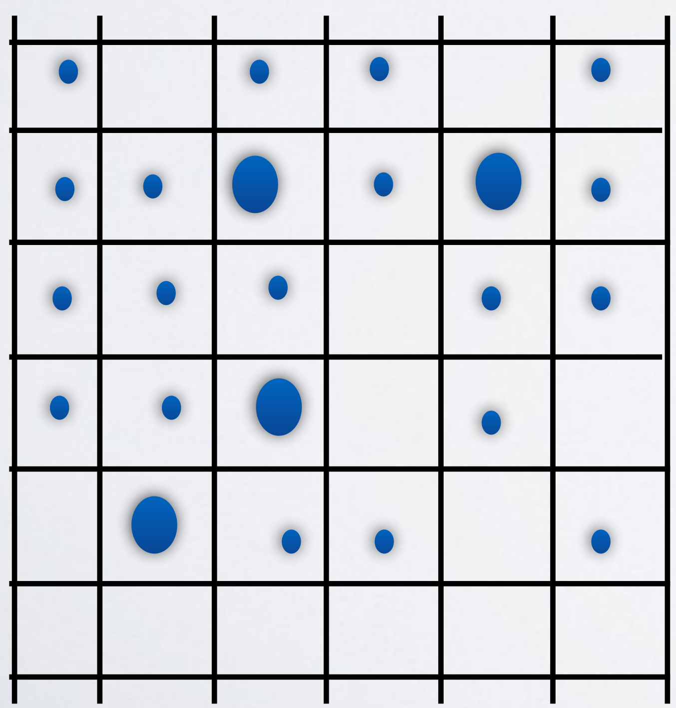
Spherical collapse at r = 0

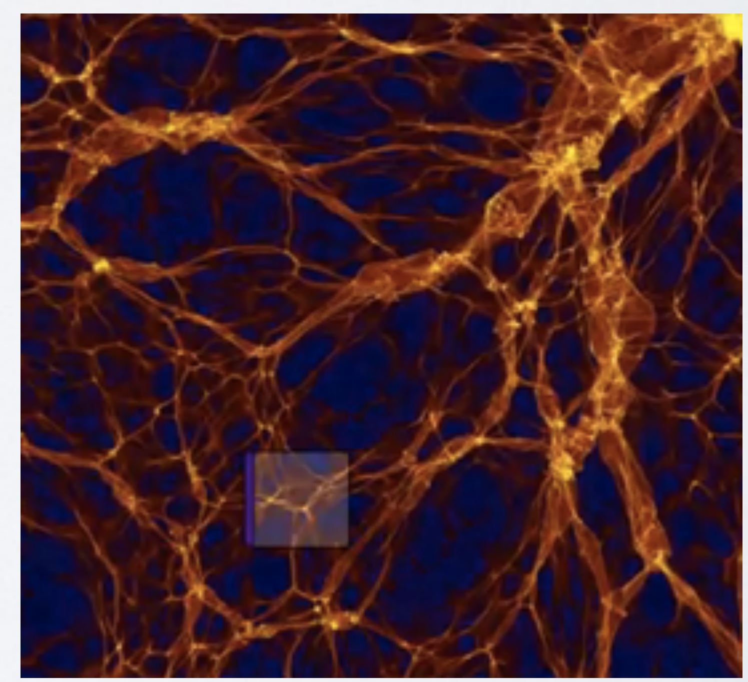
Filamentary collapse (without shell crossings) in a collapsing background

Spherical collapse at r = 0

Connection with Numerical N-body simulations

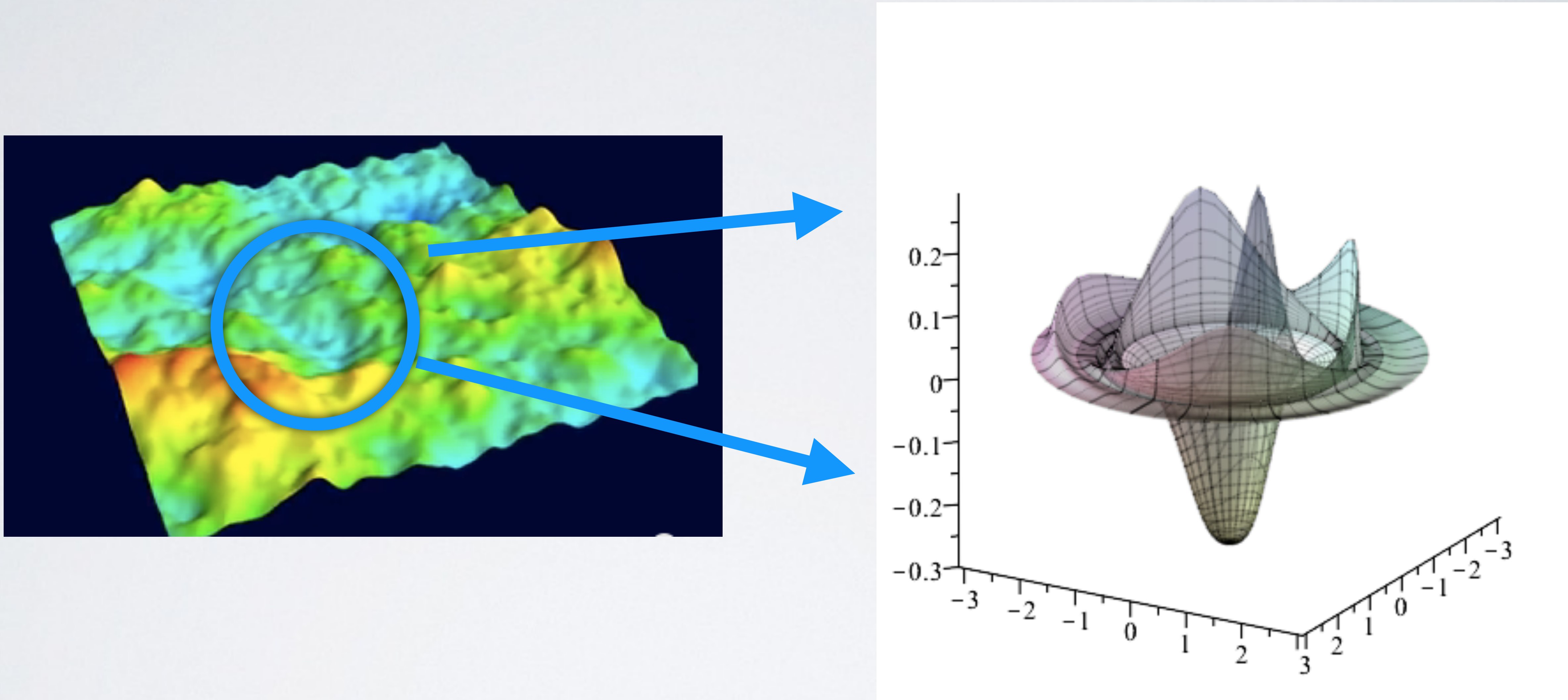




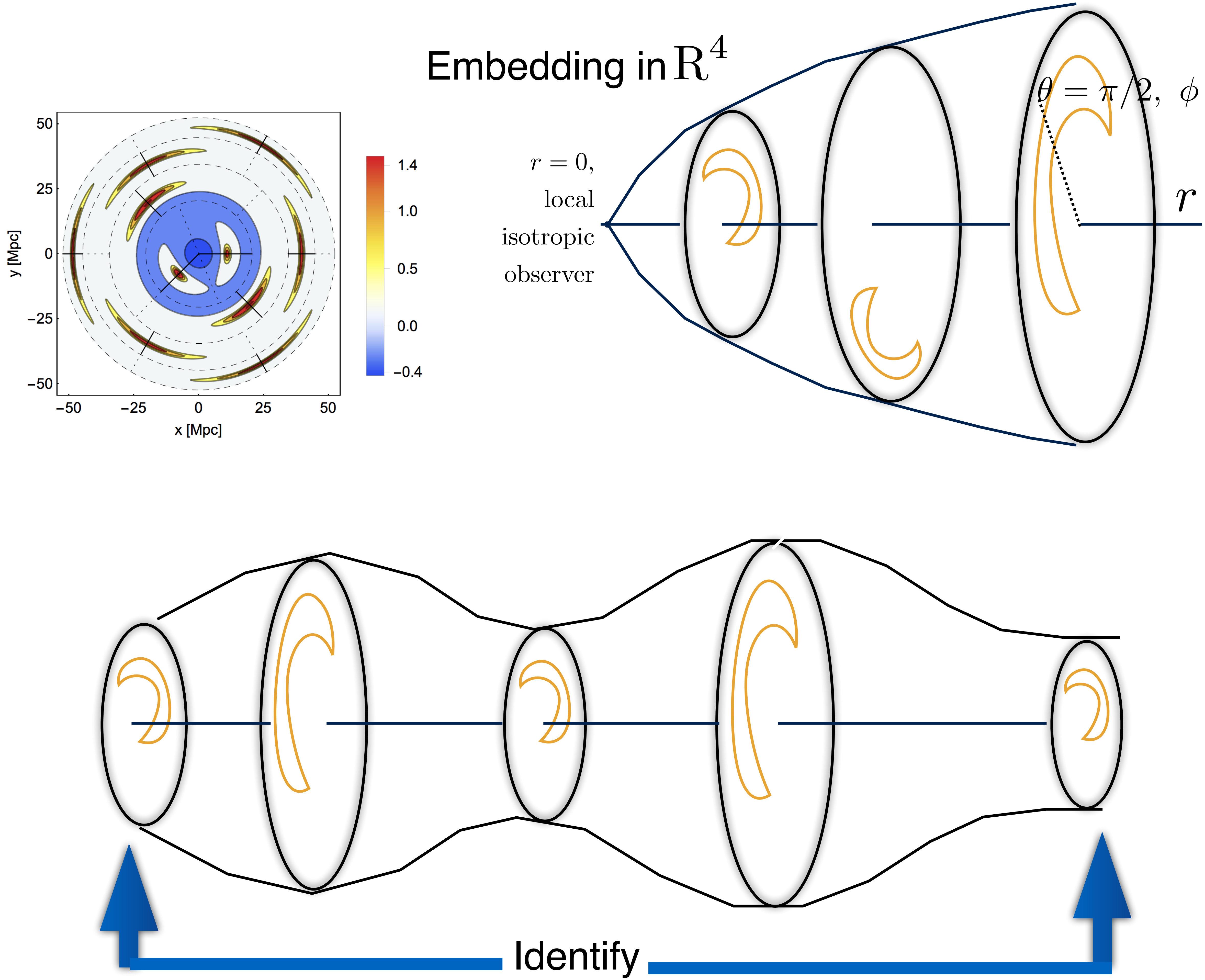


C.S. Frenk and S.D.M. White, Ann Phys, 524, 507534 (2012)

Coarse grained approximation (3d projection)



Topology & Copernican principle



Embedding in R^4
50
25
0
−25
−50
y [Mpc]
−50
−25
0
25
50
x [Mpc]
1.4
1.0
0.5
0.0
−0.4
$r = 0,$
local
isotropic
observer
$\theta = \pi/2,\ \phi$
r
Identify

Future Work

- Use Szekeres configurations as tools to test the code in numerical N-body simulations

- Compute observables, fit observations (SN, BAO, redshift distortion, CMB, Sunyaev-Zeldovich effect, etc)

- Compare with perturbative & Newtonian. Compare with alternative theories

That's all folks !