



# Habilitacja

## Boud Roukema

- **thesis**: Obserwacyjne Testy Topologii Wszechświata
- **sci**: ArFus: Galaxy Formation Modelling for the Ordinary User
- **pedagog**: Distance calculations in cosmology
- **popular**: Is the Universe curved like a sphere?

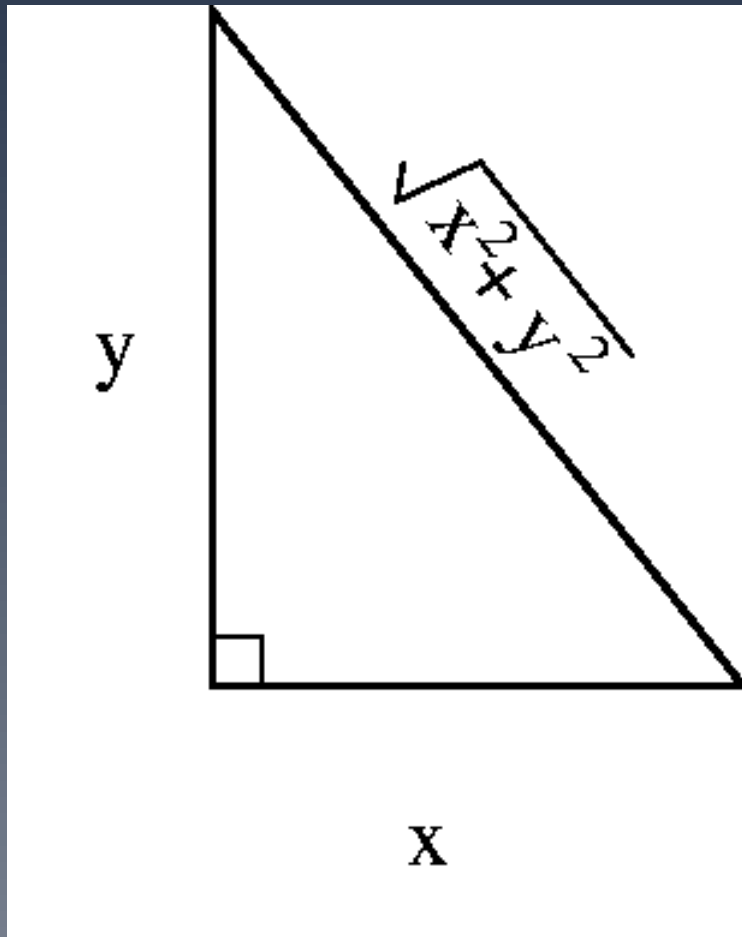


# Observational Tests of the Topology of the Universe

## Boud Roukema



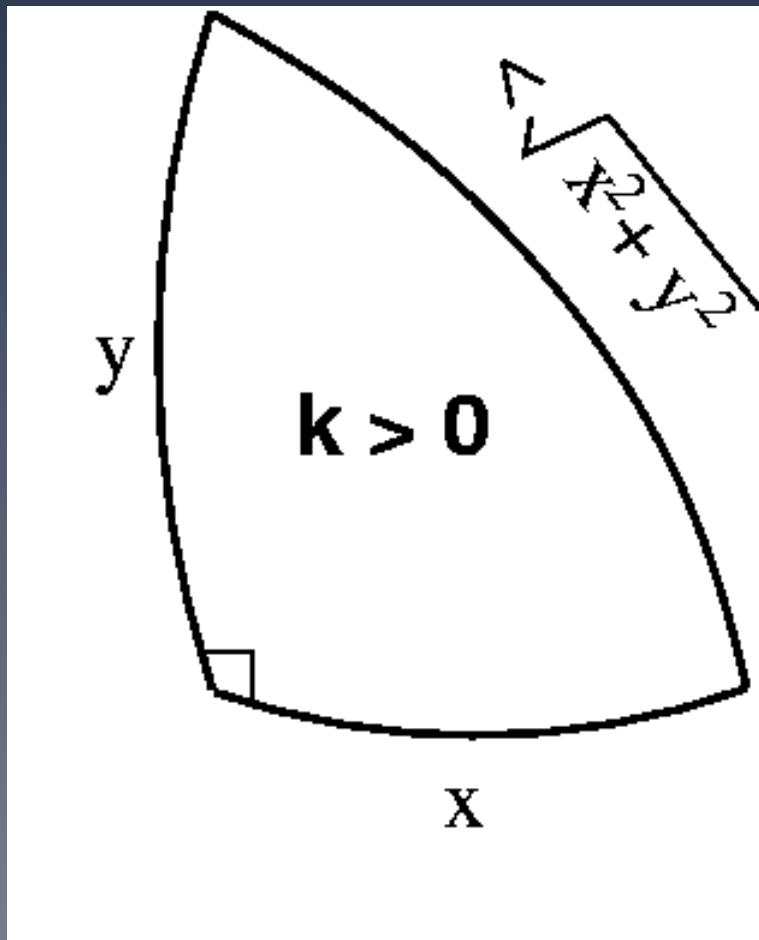
# Geometry: Curvature + Topology



0 + - multi-connected



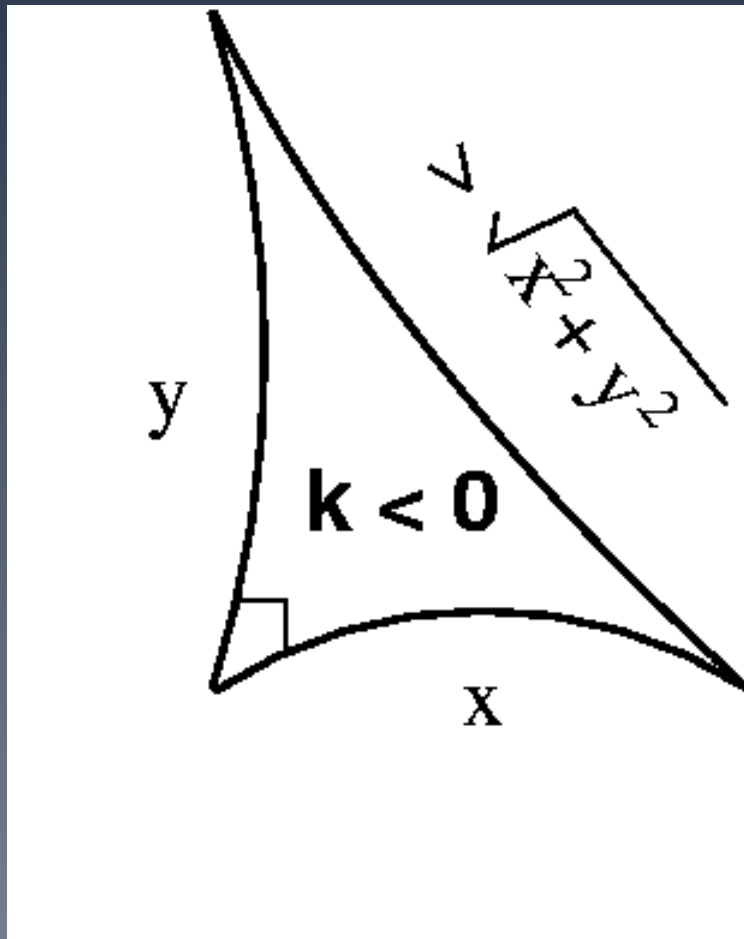
# Geometry: Curvature + Topology



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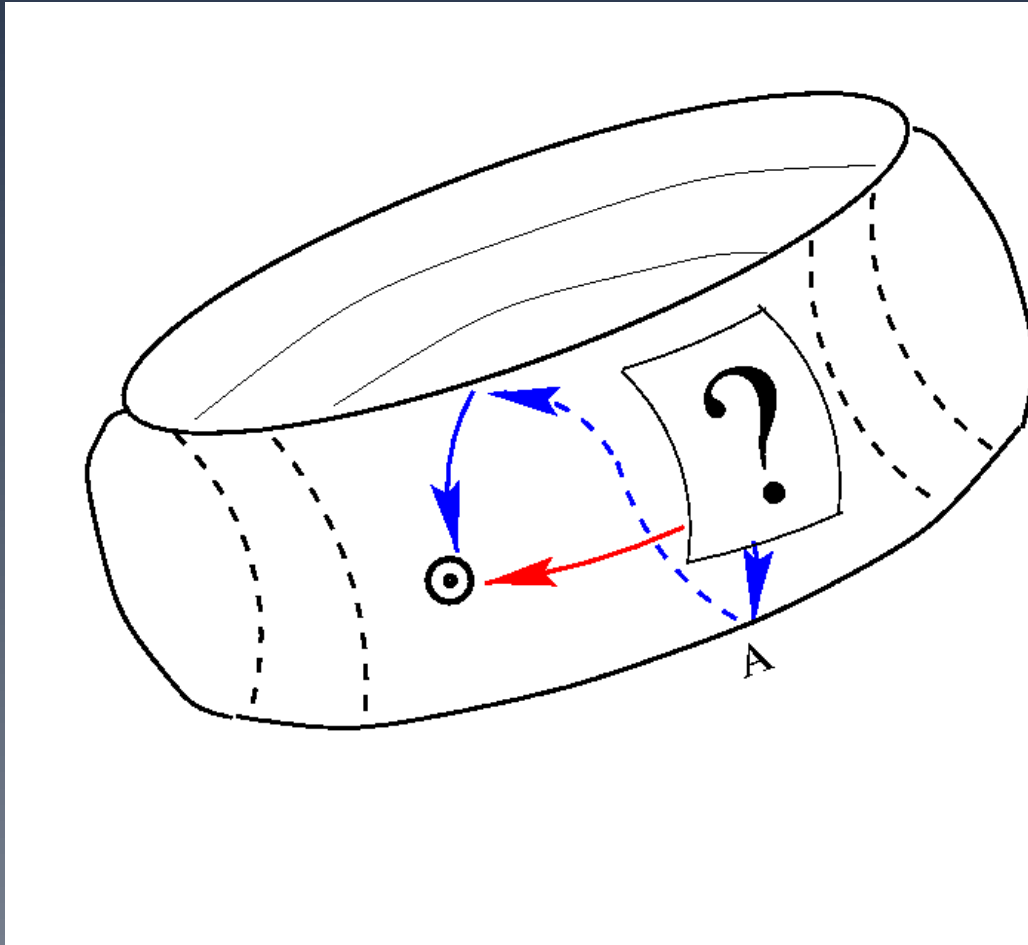
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# Quasars

Roukema B. F.

1996, Monthly Notices of the Royal Astronomical  
Society, 283, 1147

*On Determining the Topology of the Observable Universe  
via 3-D Quasar Positions*



# Clusters of Galaxies Candidate

Roukema B. F., Edge A. C. (X-ray)

1997, Monthly Notices of the Royal Astronomical Society, 292, 105

*Constraining Cosmological Topology via Highly Luminous X-ray Clusters*





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*Constraining Cosmological Topology via Highly Luminous X-ray Clusters*

Roukema B. F., Bajtlik, S. (optical)

1999, Monthly Notices of the Royal Astronomical Society, 308, 309

*Transverse Galaxy Velocities from Multiple Topological Images*



# Clusters of Galaxies Candidate

Roukema B. F. (microwave background)

2000a, Monthly Notices of the Royal Astronomical Society, 312, 712 *COBE and Global Topology: An Example of the Application of the Circles Principle*



# Application: Constraints on Curvature

Roukema B. F., Luminet, J.-P.

1999, *Astronomy & Astrophysics*, 348, 8

*Constraining Curvature Parameters via Topology*



# Cosmic Microwave Background (COBE)

Roukema B. F.

2000b, *Classical & Quantum Gravity*, 17, 3951

*A Counterexample to Claimed COBE Constraints on Compact Toroidal Models*

intro : galform : dist : pop : infl

Toruń Centre for Astronomy, UMK



# Radio-Loud Active Galactic Nuclei (RLAGNs) + Cosmic Microwave Background (WMAP)

work under progress at Toruń Centre for Astronomy,  
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- Magdalena Cechowska, Bartosz Lew (WMAP)
- <http://adjani.astro.uni.torun.pl/cosmo>





# Theory: Cosmic Topology vs Inflation

*Peaks in the Hartle-Hawking Wave Function from Sums over Topologies*

Anderson, Carlip, Ratcliffe, Surya, Tschantz, 2003

<http://arXiv.org/abs/gr-qc/0310002>

- some topologies are much more probable than others
- spatial metrics of constant (negative) curvature are favoured
- work incomplete, but hints at predictability



# Fine-Tuning

- observable  $\Omega_\Lambda > 0 \Rightarrow$  fine-tuning of inflation
- observable cosmic topology  $\Rightarrow$  fine-tuning of inflation
- both might be the result of the same fine-tuning of inflation, or else of some other mechanism (e.g. peak in Hartle-Hawking wave function from sums over topologies)



# ArFus: Galaxy Formation Software for the Ordinary User

(printed transparencies)



# Distance calculations in cosmology

- light-travel distance:

$$d_{\text{light-travel}} = \int_t^{t_0} c dt' \quad (1)$$



proper distance = comoving distance =

$$\chi = \int_t^{t_0} \frac{c dt'}{a(t')}$$



proper distance = comoving distance =

$$\begin{aligned}\chi &= \int_t^{t_0} \frac{c \, dt'}{a(t')} \\ &= \frac{c}{H_0} \int_{1/(1+z)}^1 \frac{da}{a \sqrt{\Omega_m/a - \Omega_\kappa + \Omega_\Lambda a^2}}\end{aligned}\quad (2)$$

[http://www.wikipedia.org/wiki/Comoving\\_coordinates](http://www.wikipedia.org/wiki/Comoving_coordinates)



proper motion distance = coordinate distance =

$$d_{\text{pm}} = \begin{cases} R_C \sinh \frac{\chi}{R_C} & k = -1 \\ \chi & k = 0 \\ R_C \sin \frac{\chi}{R_C} & k = +1 \end{cases} \quad (3)$$



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$$d_L = (1 + z)d_{\text{pm}} = (1 + z)^2 d_a \quad (4)$$





# FLRW metric

$$ds^2 = c^2 dt^2 - a^2(t) [d\chi^2 + dd_{\text{pm}}^2 (d\theta^2 + \cos^2 \theta d\phi^2)] \quad (5)$$



# Non-radial spatial geodesics

What is the comoving distance between two objects at different celestial positions and different redshifts, for an arbitrary curvature  $0 + - ?$



# Distances on the 2-sphere

$$\begin{aligned}x_i &= R \cos \delta_i \cos \alpha_i \\y_i &= R \cos \delta_i \sin \alpha_i \\w_i &= R \sin \delta_i\end{aligned}\tag{6}$$



# Distances on the 2-sphere

$$\begin{aligned}x_i &= R \cos \delta_i \cos \alpha_i \\y_i &= R \cos \delta_i \sin \alpha_i \\w_i &= R \sin \delta_i\end{aligned}\tag{6}$$

$$\langle a_1, a_1 \rangle = x_1 x_2 + y_1 y_2 + w_1 w_2\tag{7}$$

(cf 11, 13)

intro : galform : dist : pop : infl

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but also:

$$\langle a_1, a_1 \rangle = R^2 \cos \theta_{12}. \quad (8)$$



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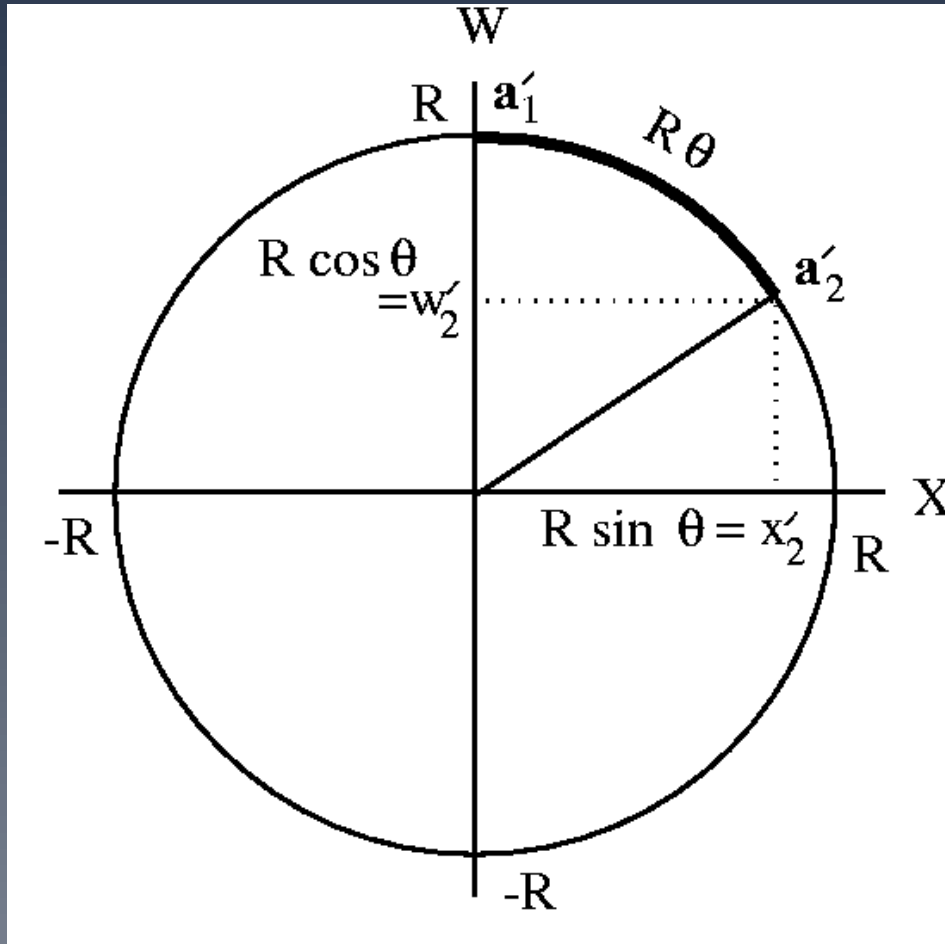
a distance in  $\mathcal{S}^2 =$  arc-length in  $\mathcal{R}^3$ :

$$\chi_{12} = R \theta_{12} = R \cos^{-1} [\langle a_1, a_2 \rangle / R^2]. \quad (9)$$

(cf 14)

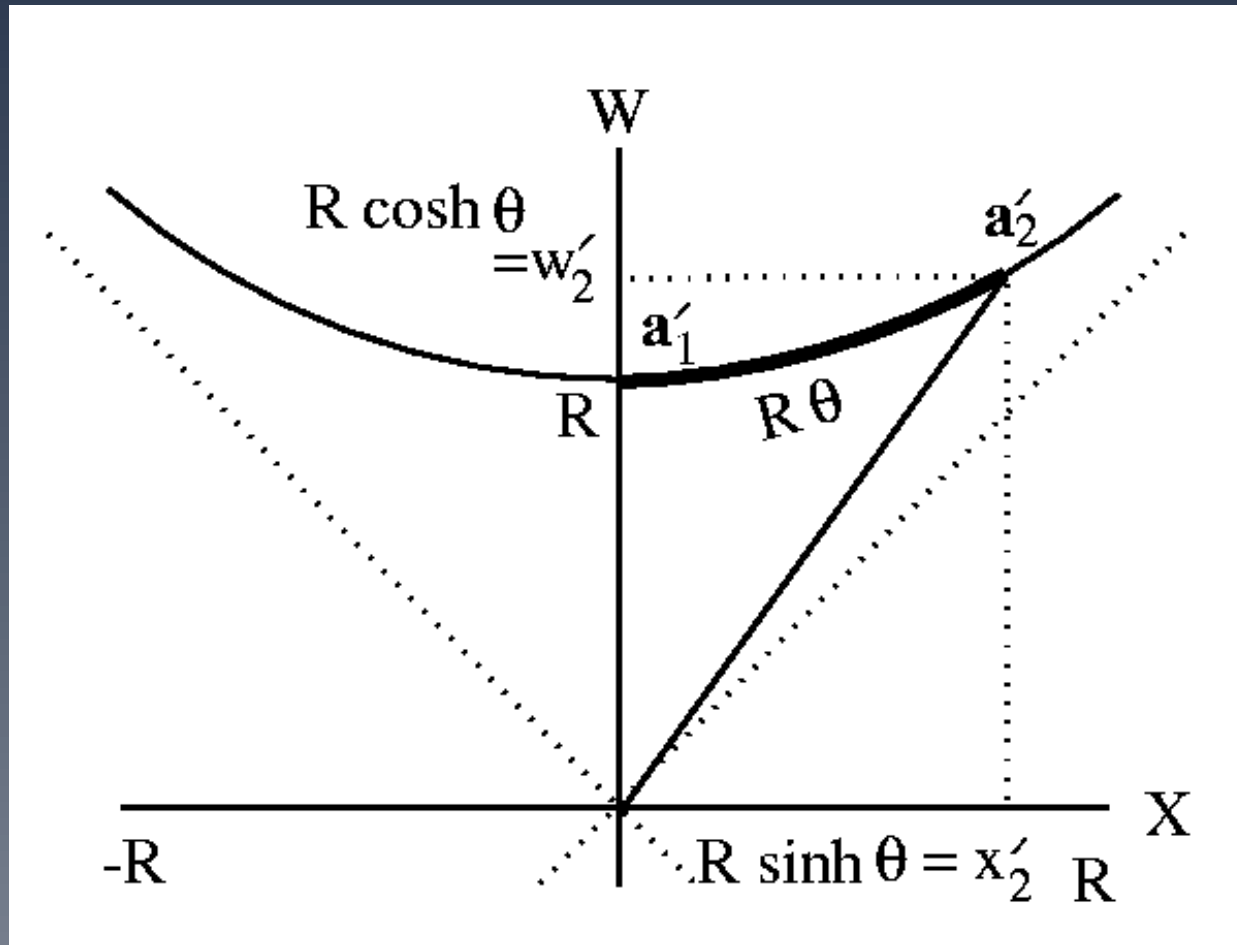


# positive curvature





# negative curvature







## Distances on the 3-sphere (3-hyperboloid)

$$\Sigma(\chi_i) \equiv \begin{cases} R \sinh(\chi_i/R) & k \equiv -1 \\ \chi_i & k \equiv 0 \\ R \sin(\chi_i/R) & k \equiv +1 \end{cases} \quad (10)$$

$$x_i = \Sigma(\chi_i) \cos \delta_i \cos \alpha_i$$

$$y_i = \Sigma(\chi_i) \cos \delta_i \sin \alpha_i$$

$$z_i = \Sigma(\chi_i) \sin \delta_i$$

$$w_i = \begin{cases} R \cosh(\chi_i/R) & k = -1 \\ 0 & k = 0 \\ R \cos(\chi_i/R) & k = +1 \end{cases} \quad (\text{cf eq. (6)})(11)$$



metric on  $\mathcal{S}^3$  (or  $\mathcal{R}^3$  or  $\mathcal{H}^3$ ):

$$ds^2 = \begin{cases} k(dx^2 + dy^2 + dz^2) + dw^2 & k = \pm 1 \\ dx^2 + dy^2 + dz^2 & k = 0. \end{cases} \quad (12)$$

inner product (cf 7):

$$\langle a_1, a_2 \rangle \equiv \begin{cases} k(x_1x_2 + y_1y_2 + z_1z_2) + w_1w_2 & k = \pm 1 \\ x_1x_2 + y_1y_2 + z_1z_2 & k = 0. \end{cases} \quad (13)$$



generalisation of eq. (9):

$$\chi_{12} = \begin{cases} R \cosh^{-1} [\langle a_1, a_2 \rangle / R^2] & k = -1 \\ \sqrt{\langle a_1 - a_2, a_1 - a_2 \rangle} & k = 0 \\ R \cos^{-1} [\langle a_1, a_2 \rangle / R^2] & k = +1. \end{cases} \quad (14)$$

a distance in  $\mathcal{S}^3$  is an arc-length in  $\mathcal{R}^4$



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a distance in  $\mathcal{H}^3$  is an arc-length in  $\mathcal{M}^4$

<http://arXiv.org/abs/astro-ph/0102099>

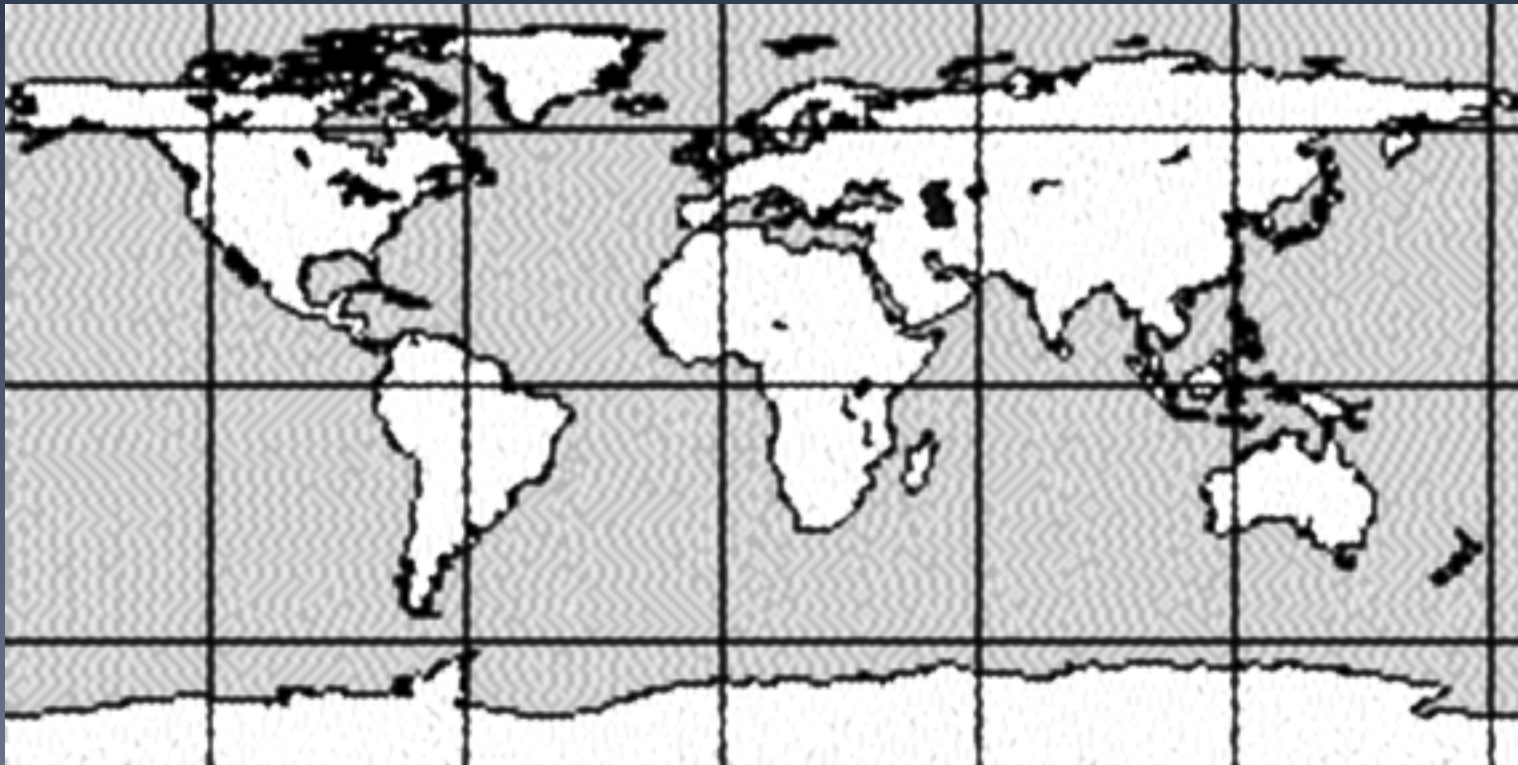


# Is the Universe curved like a sphere?

- How can we think of curvature?
- How can we measure curvature?
- Finding a standard ruler
- Using a standard ruler - **LSS**

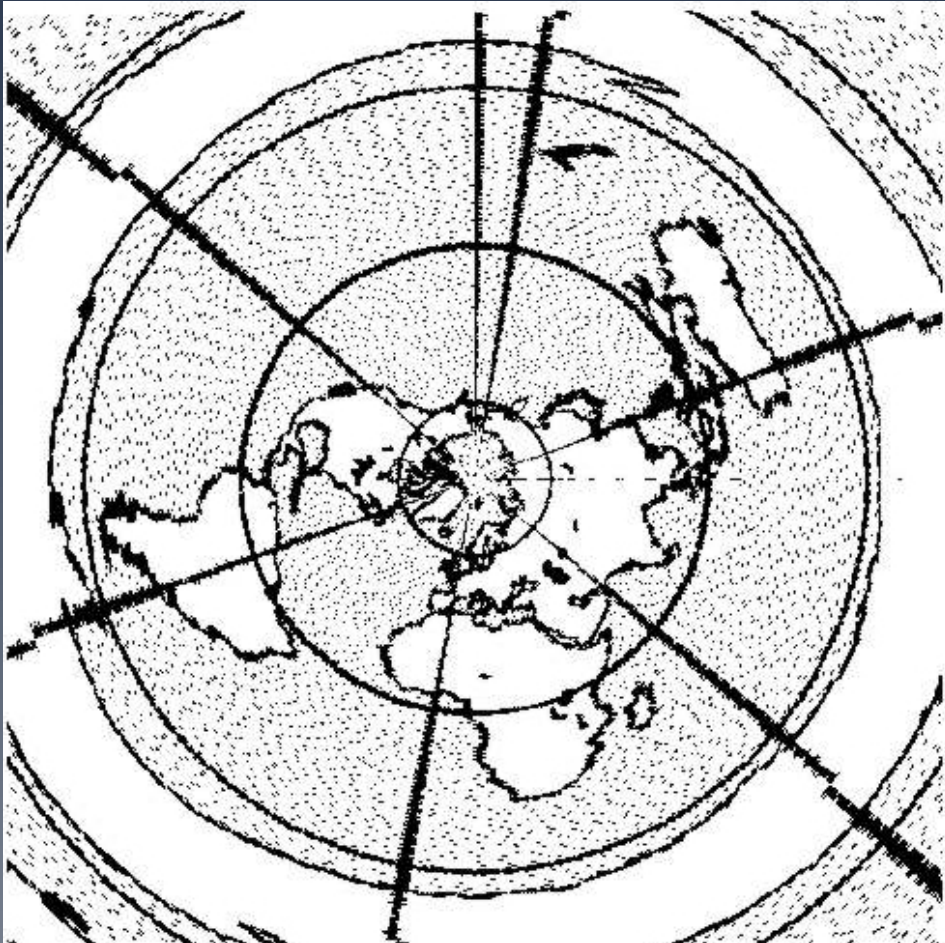


# A flat Earth?



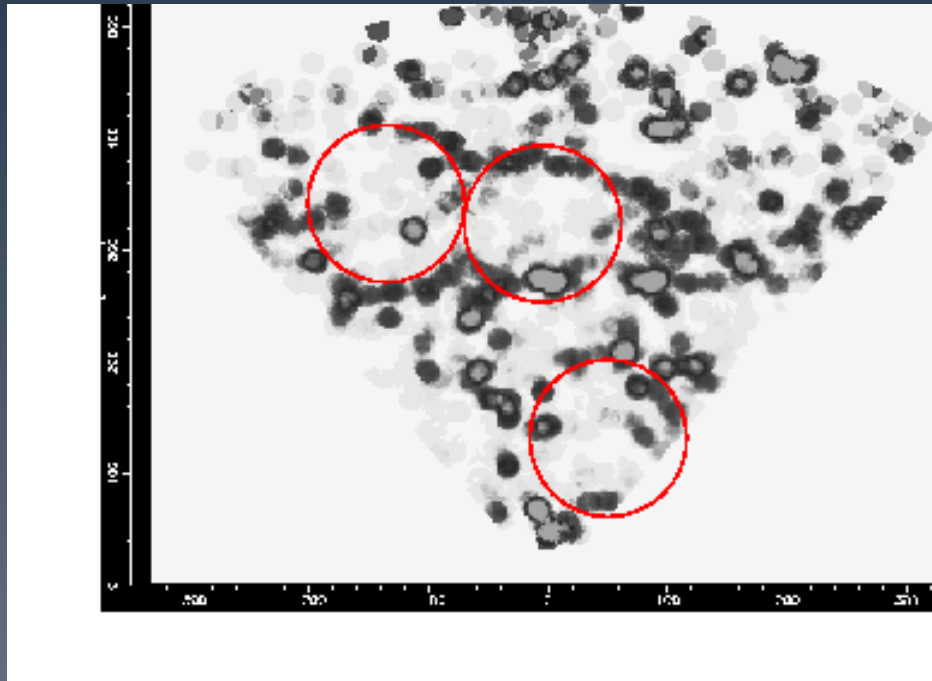


# Another flat Earth?





# A Standard Ruler: LS Structure Bubbles

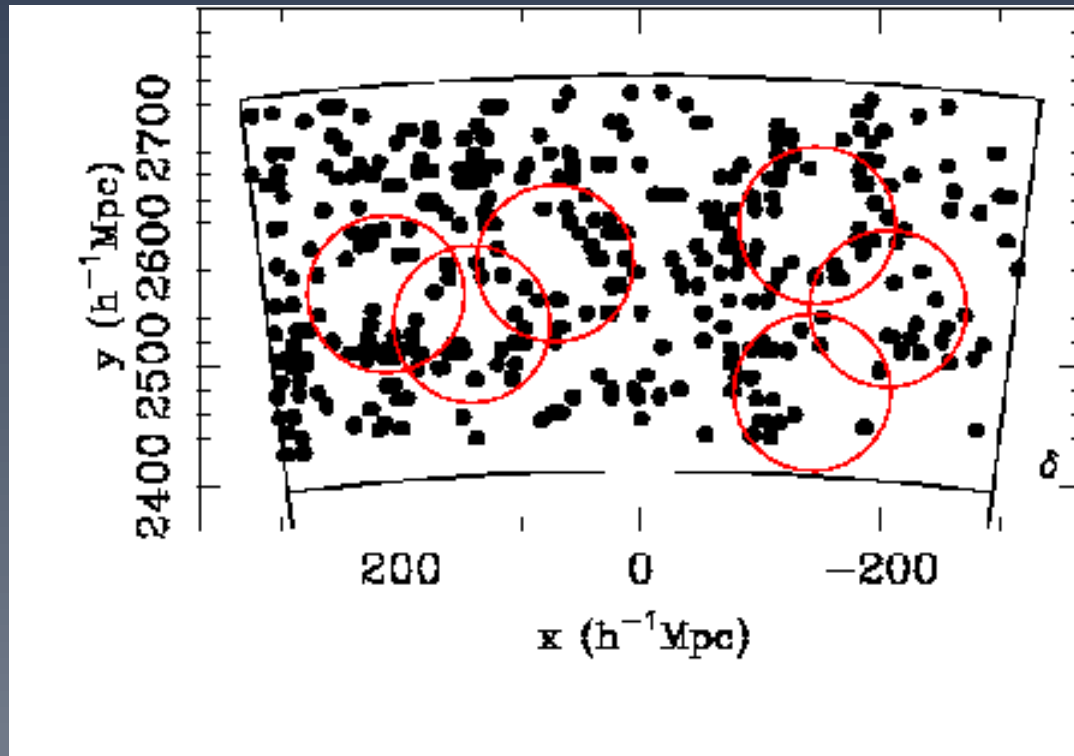


$z=0$   $\Lambda=0$   $\Lambda$





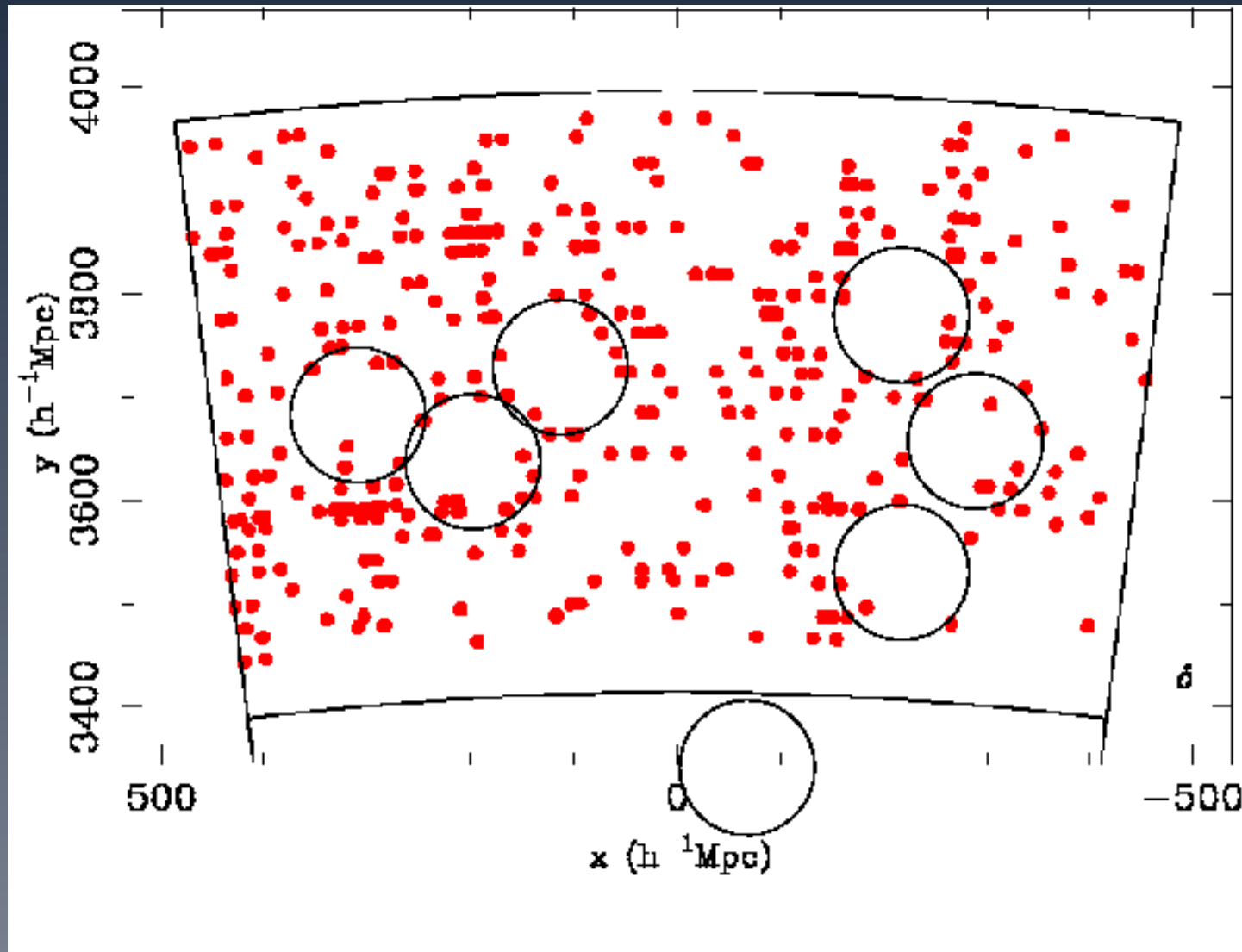
# Distant quasars: flat (NO cosmological constant)



$z=0$   $\Lambda=0$   $\Lambda$



# Distant quasars: flat WITH cosm constant



$z_0 \Lambda_0 \Lambda$



# Conclusion

Using the large scale structure “bubbles” traced by galaxies and quasars as a **standard ruler**, distant structures match nearby structures best if the Universe is approximately **flat** with about **70% of matter-energy density** in a **cosmological constant**.