Example of an inhomogeneous cosmological model inspired by the perturbation theory

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Motivation:

We were looking for a simple inhomogeneous cosmological model, with a metric given explicitly, for which one can apply the Green-Wald scheme and Buchert averaging technique simultanously.

The metric

$$g_{\mu\nu} = g^{(0)}_{\mu\nu} + \lambda\, h_{\mu\nu}$$

The background is the Einstein-de Sitter. In the cartesian coordinates (t, x, y, z):

$$g^{(0)}_{\mu\nu} = \mathrm{diag}(-1, a^2, a^2, a^2), \quad a(t) = \mathcal{C}\, t^{2/3}, \quad \mathcal{C} = \mathrm{const.}$$

The perturbation has the following form:

$$h_{00} = 0, \quad h_{i0} = 0, \quad h_{ij} = a^2 \left(C_{,ij} - \frac{1}{3}\delta_{ij}(C_{,xx} + C_{,yy} + C_{,zz}) + \delta_{ij}\, D \right)$$

$$C(t,x,y,z) = -\frac{\mathcal{C}^3\lambda}{81t}\left(\frac{x^2}{16} + \frac{y^2}{16} + \frac{z^2}{16} + \frac{1}{32B^2}\cos(2Bx) + \frac{1}{32B^2}\cos(2By) + \frac{1}{32B^2}\cos(2Bz)\right)$$

$$D(t,x,y,z) = -\frac{\mathcal{C}^3\lambda}{243t}\left(\frac{1}{8}(-\cos(2Bx)+1) + \frac{1}{8}(-\cos(2By)+1) + \frac{1}{8}(-\cos(2Bz)+1)\right)$$

The metric

$$g_{\mu\nu} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & \mathcal{C}^2 \sqrt[3]{t} \left(- \frac{\mathcal{C}^3 \lambda}{324} \sin^2 (Bx) + t \right) & 0 & 0 \\ 0 & 0 & \mathcal{C}^2 \sqrt[3]{t} \left(- \frac{\mathcal{C}^3 \lambda}{324} \sin^2 (By) + t \right) & 0 \\ 0 & 0 & 0 & \mathcal{C}^2 \sqrt[3]{t} \left(- \frac{\mathcal{C}^3 \lambda}{324} \sin^2 (Bz) + t \right. \end{bmatrix}$$

The constant $\mathcal{C}$ is determined by the condition $a(t_0) = 1$,

where t_0 is the age of the Einstein-de Sitter universe with a given H_0

The energy-momentum tensor

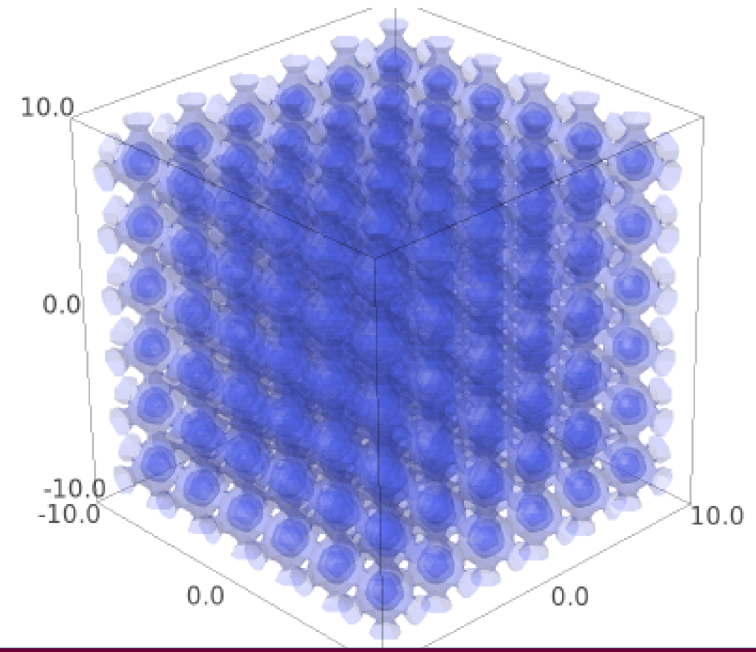
$$G_{\mu\nu} = G^{(0)}_{\mu\nu} + \lambda\, G^{(1)}_{\mu\nu} + \dots$$

$$T_{\mu\nu} = T^{(0)}_{\mu\nu} + \lambda\, T^{(1)}_{\mu\nu} + \dots$$

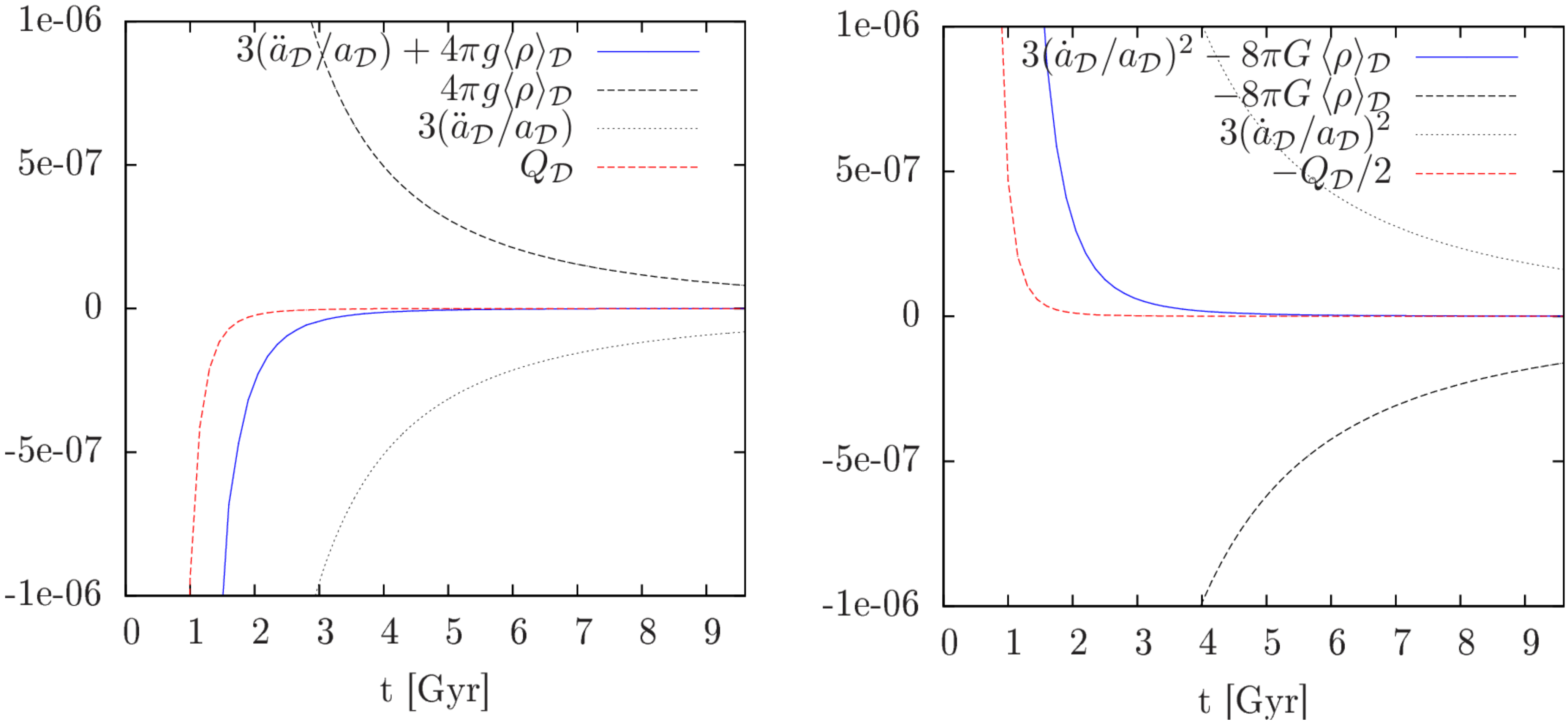
$$T^{(k)}_{\mu\nu} = G^{(k)}_{\mu\nu}/8\pi$$

$$T^{(0)}_{\mu\nu} = \rho^{(0)}\, U_\mu\, U_\nu, \quad U^\mu = (1,0,0,0), \quad \rho^{(0)} = \frac{4}{3}t^{-2}$$

$$T^{(1)}_{\mu\nu} = \rho^{(1)}\, U_\mu\, U_\nu, \quad \rho(1) = \frac{1}{3888\pi t^3}\left(\mathcal{C}^3 \sin^2(Bx) + \mathcal{C}^3 \sin^2(By) + \mathcal{C}^3 \sin^2(Bz)\right)$$

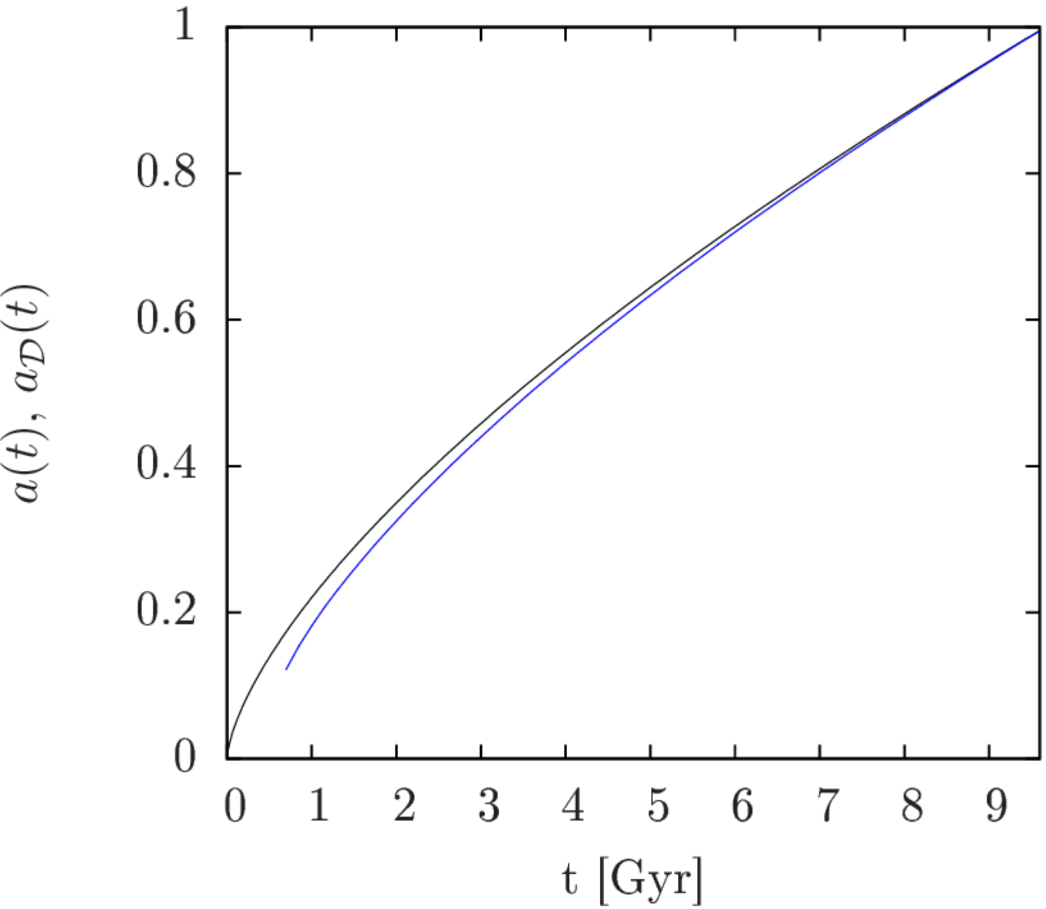


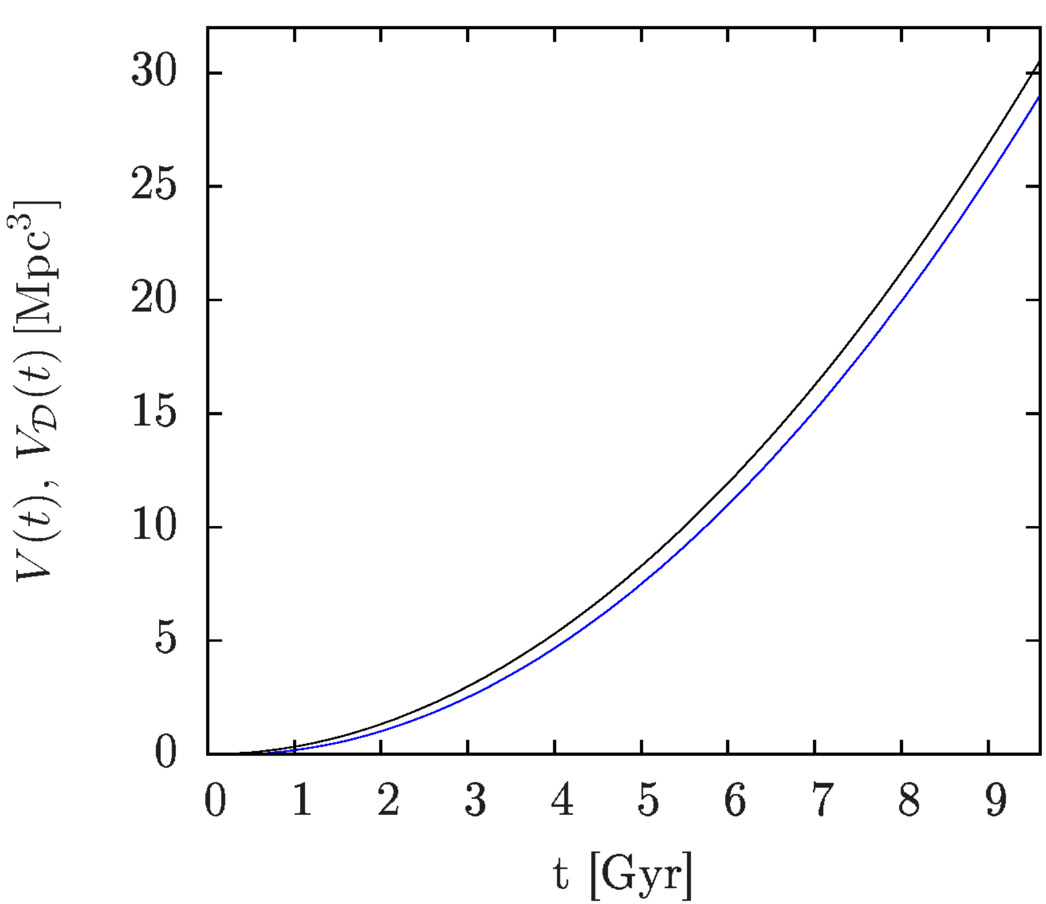
When the higher order terms are small?



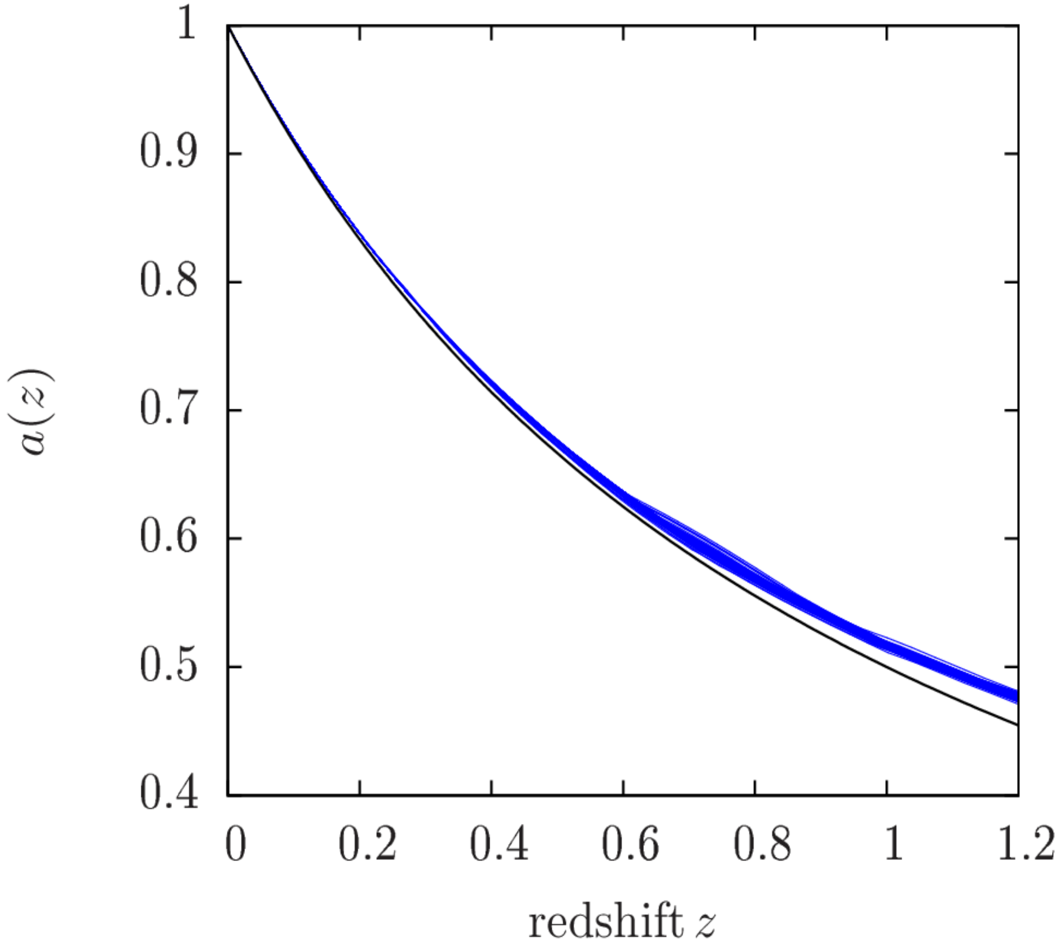
We fixed the scale parameter $B = 1$,
and the amplitude λ so that $\langle\rho^{(0)}\rangle_{\mathcal{D}}$ is 0.04 in critical units

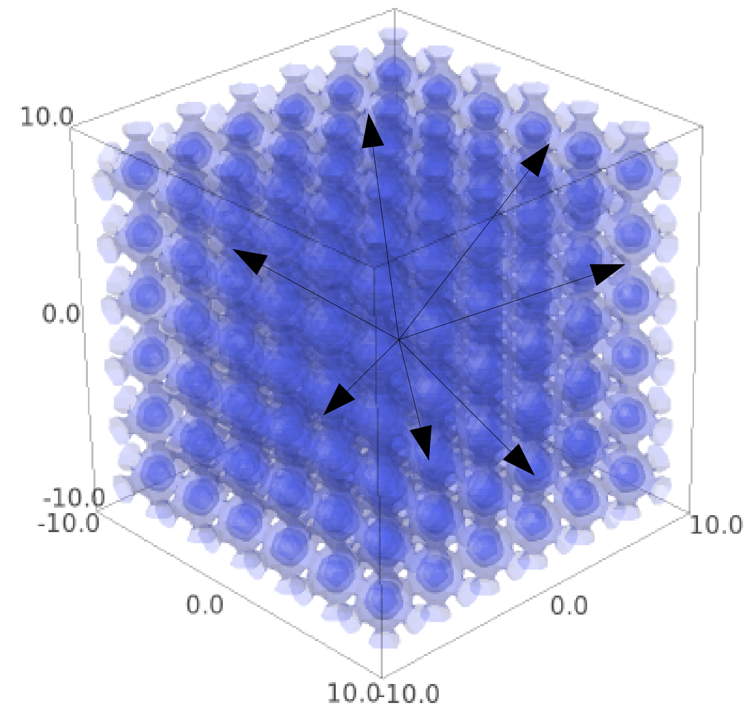
Effective scale factor and the volume of the elementary cell





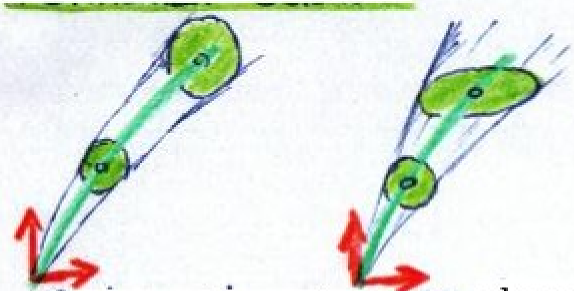
The null geodesics





The angular diameter distance

Sachs formalism

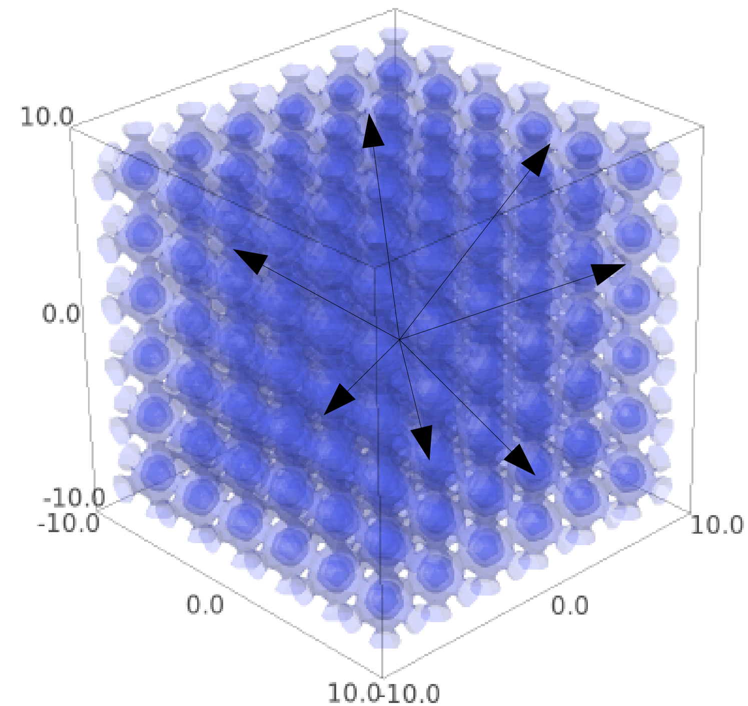


$$\begin{cases} \theta = \frac{1}{d_A}\frac{d}{d\lambda} d_A \\ \sigma = \sigma_1 + i\sigma_2 \end{cases}$$

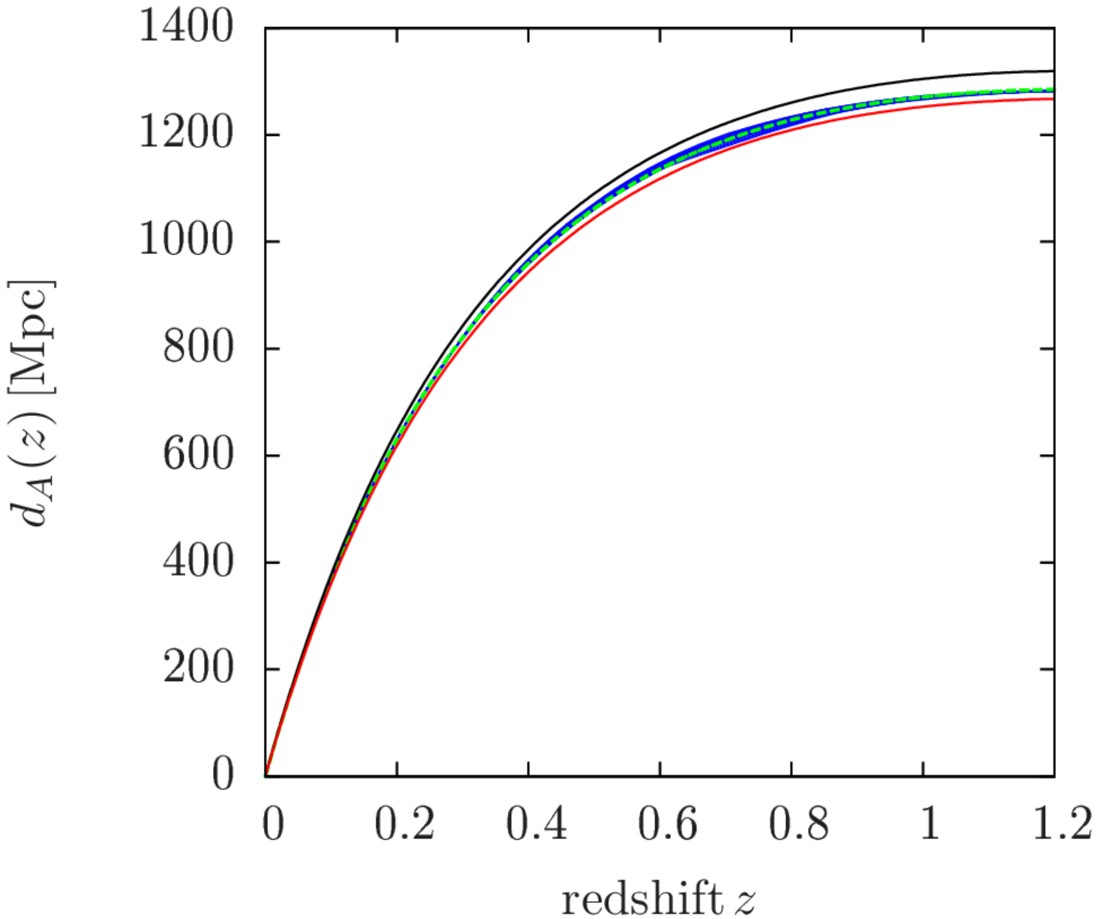
θ · expansion scalar σ - shear

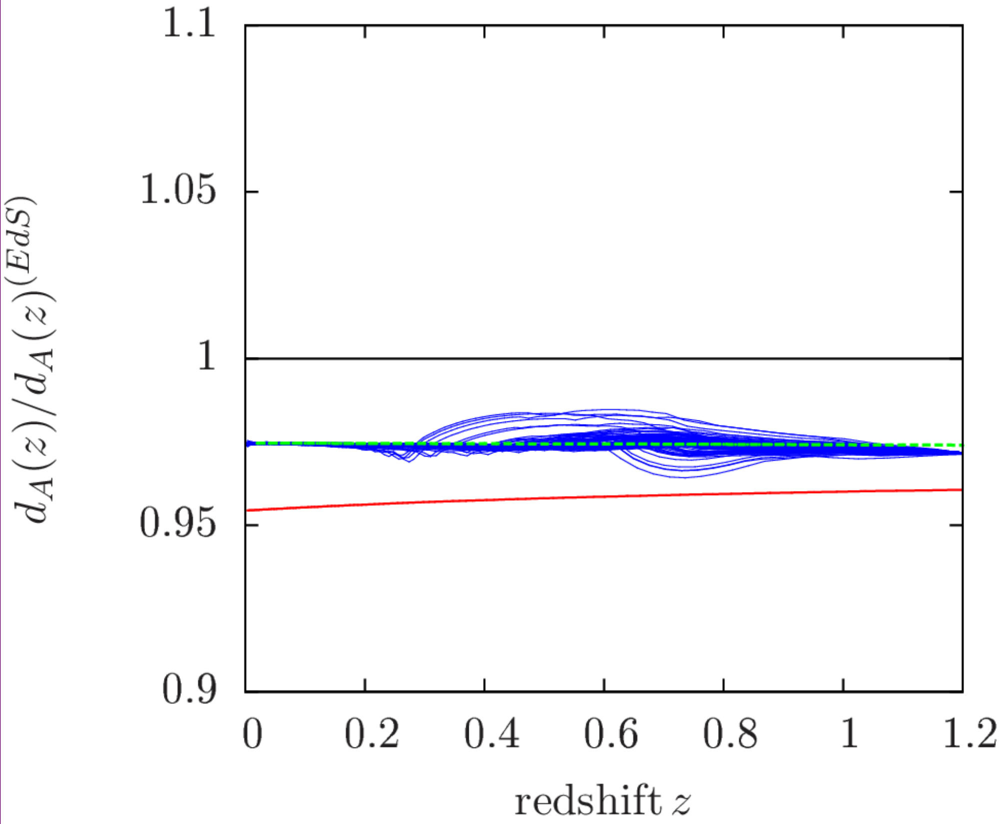
$$\begin{cases} \frac{d^2}{d\lambda^2} d_A = -\left(\frac{1}{2} R_{\mu\nu} k^\mu k^\nu + |\sigma|^2\right) d_A \\ \frac{d}{d\lambda}\sigma_1 + 2\sigma_1\theta = -\frac{1}{2} C_{\alpha\beta\gamma\delta}\left(s_1^\alpha k^\beta k^\gamma s_1^\delta + s_2^\alpha k^\beta k^\gamma s_2^\delta\right) \\ \frac{d}{d\lambda}\sigma_2 + 2\sigma_2\theta = C_{\alpha\beta\gamma\delta}\left(s_1^\alpha k^\beta k^\gamma s_2^\delta\right) \end{cases}$$

initial condition : $\lambda = 0$: $d_A \sim 10^{-6}$ $\dot{d}_A = 1$ $\sigma_1 = 0$ $\sigma_2 = 0$



The angular diameter distance





Thank you for your attention.

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Symbolic calculations has been done with the help of the CAS: ***Maxima*** and ***Mathematica****:*